PEER-GROUP FORECASTING INCENTIVES FOR UNOBSERVED VARIABLES

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NOTICE OF PATENT ISSUED

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This paper describes a method of economic incentives, involving plural-forecaster payment systems, upon which the author and inventor has been issued a patent. (U.S. Patent 5,608,620) The patent on this invention only restricts actual use of the described invention; it does not restrict in any way the verbal or written discussion, description, or criticism of that invention. Any actual use of the patented invention without the express written permission of Forecasting Patents, Inc. is strictly prohibited.

ABSTRACT

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A key feature of some variables of interest is that they are never accurately observed. This paper describes how an incentive scheme which pays forecasters in accordance with their value marginal product (VMP) can be used to elicit unbiased forecasts or estimates of unobserved variables from individual forecasters. The peer-group nature of the incentive scheme arises because predictions are judged only by other predictions, not by observed events. Multiple Nash equilibria are possible. Truthful Nash equilibrium emerges if a) truth-telling is a focal point for equilibrium, or b) some forecasters are truthful regardless of incentive.

J.E.L. Codes: D80 Information and Uncertainty
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D62 Externalities
H23 Externalities

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INTRODUCTION[1]

A key feature of some variables of interest is that they are not precisely observed, which makes it difficult to assess the accuracy of estimates or forecasts provided by expert prognosticators. This paper focuses on finding a useful set of incentives for eliciting accurate estimates or predictions of variable values which are never precisely observed.

To this problem I apply a method of incentives for eliciting forecasts first described in Lundgren (1994). This method of forecasting incentives attempts to pay each forecaster in accordance with a measure of the value marginal product (VMP) of the forecaster's contribution to the accuracy of a collective prediction. The method of computing payment is relatively straightforward if the variable value of interest is expected to be observed in the future with some precision. The future observation can then be used to reward forecasters in accordance with how well their predictions contributed to the accuracy of a collective forecast.

For some variables (e.g., environmental damages, G.N.P. in 2025), a precise observation of variable values is never to be expected, or is not to be expected any time soon. In such a circumstance, something other than a future observation of the variable in question must be used as a criterion value for determining the accuracy or inaccuracy of forecasters' estimates or predictions. This paper describes a technique for using the predictions of other forecasters to compute the needed criterion value for use in a VMP forecasting incentive method. Because the technique uses only the simultaneous predictions of other forecasters, rather than actual observations, to determine forecaster rewards, the technique may be referred to as a "peer-group" method, since the forecasts of one's peers are used to determine the marginal value of one's forecasts.

Previous literature on forecasting incentives includes Osband (1989, 1985), Kadane and Winkler (1988), and Page (1988). These papers are further described in Lundgren (1994). Numerous papers explore the forecasting incentives which appear implicit in stock markets, bond markets, and futures commodity markets.[2] Aside from such analyses, there appears to be very little literature on alternative forecasting incentive schemes, and none which relates to incentives for eliciting unbiased estimation of unobservable variable values.

Section I provides some examples of unobserved variables. Section II discusses the VMP forecasting incentive method. Section III provides a base-case example using precision weights, under the assumption that the predicted variable value is later observed. Section IV describes a modification of the VMP technique which allows for the elicitation of predictions for unobservable variable values. Section V shows that truth-telling is an equilibrium of the modified incentive scheme, but that other equilibria also exist. Section VI considers whether non-optimizing ideological extremists are likely to bias the equilibrium outcome. Section VII explores the relationship between the expected pay of forecasters and their VMP under the modified incentive scheme. Section VIII concludes.

I. Examples of Unobserved Variables

One field of interest where unobserved variables can be found in profusion is that of the environment. Environmental externalities, such as air pollution or water pollution, come from many different sources, come in many different forms, and can have a variety of effects.

If a pollution tax is to be imposed, an estimate of the marginal damage caused by each type of pollutant is necessary to determine the appropriate tax rate for each pollutant. Values for pollution damage are unobserved for at least two reasons. First, because the causal relationships between pollution levels and specific physical effects may be scientifically uncertain. Second, because the translation of physical damages into monetary damages may likewise be uncertain, even if one adopts a particular economic methodology (e.g., willingness to pay based on consumer surplus). The use of common-sense and expert judgement of all types--scientific, economic, econometric--is needed to arrive at an assessment of marginal damages. The judgements of reasonable people may very well differ, and there is generally no observable objective measure to determine which judgement is most reasonable.

Unobserved variables are important in other fields as well. For example, one may desire knowledge of the effectiveness of law enforcement efforts by estimating the quantity and severity of unreported crime, illegal migration, drug smuggling, pollution control evasion, or tax evasion. Or one may desire to estimate the effects of a change in regulation or government expenditure on health and safety, tax revenues, consumer surplus, or social behaviors. If the change is not implemented, the effect of the change cannot be observed. Even if the change is implemented, its true effects may not be entirely discernible, due to additional factors which influence the same effects.

Even where variable values are technically observable, there may be circumstances where a peer-group forecasting incentive scheme may be useful. For example, the variable value may not be observable for some considerable period of time (perhaps several years or decades), making it less useful or not useful for calculating forecaster compensation. Or the variable might have a high *ex post* variance relative to forecasters' *ex ante* knowledge, such as an estimate of the probability of an unlikely event. Risk-averse forecasters may require less compensation to participate in an incentive scheme which imposes less risk, and may thereby favor an incentive scheme which treats a particular variable as if it were unobservable.

II. Background on VMP Forecasting Incentives

The VMP forecasting incentives technique assumes that there is a principal (forecast requisitioner) whose goal is to obtain an accurate prediction concerning the future realization of a random variable X. This goal is to be accomplished indirectly, rather than directly, by hiring a set of agents (forecasters) who will do the actual forecasting. The problem for the forecast requisitioner is to find a set of contracts for the forecasters such that the incentives given to the forecasters result in tolerably good forecasts at a tolerably low cost. Due to limited powers of observation, the principal cannot condition the parameters of the incentive contracts on any detailed knowledge of how the forecasters perform their task.

The forecast requisitioner aggregates the predictions of individual forecasters to obtain a collective prediction suitable for further action. A typical method of aggregation might be to take an average or weighted average of forecasters' predictions, such as an arithmetic mean or a geometric mean.[3] Let X_c represent the vector of individual predictions, X_1, X_2, ..., X_n of forecasters 1, 2, ..., n. A prediction aggregator function can be generalized as follows:

$$G(X_c) = G(X_1,X_2,X_3,...,X_n) \qquad (1)$$

Let X_{ci} represent the vector of predictions of all forecasters except forecaster i. $G(X_{ci})$ represents a "secondary collective prediction," which would presumably be issued in the absence of forecaster i's prediction.

Define $B(X_a,G(X_c))$ as the benefits which accrue when $G(X_c)$ is the collective prediction of X, while X_a is an actual or estimated value of X which is later observed. The loss function, $L(X_a,G(X_c))$, tells us the lost benefits which occur when the predicted X differs from its actual value:

$$L(X_a,G(X_c)) = B(X_a,X_a) - B(X_a,G(X_c)) \qquad (2)$$

The value X_a can be used as a "criterion value"--a variable value which is used to judge the accuracy or inaccuracy of forecasters' predictions. If the actual value of the variable being predicted is observed within a reasonable period of time, it is natural to use the actual variable value as the criterion value. Otherwise, it will be necessary to use a proxy (for example, when estimating environmental externalities).

The payment schedule for each forecaster can be made a function of X_a and each X_i: $P_i=P_i(X_a,X_i,X_{ci})$. Given the payment schedule, each forecaster will choose his prediction to maximize his own utility, given his own utility function which we may presume is not directly observed by others. The VMP incentive scheme attempts to measure forecaster VMP and pay in accordance therewith. In Lundgren (1994) it is shown that the VMP incentive scheme has various favorable properties. For instance, the scheme results in optimal effort levels by each forecaster, and attracts nearly optimal numbers of forecasters to the forecasting task.

The VMP incentive method uses a proxy for VMP which may be termed "marginal contribution." The marginal contribution asks how the value of a collective forecast changes, when the prediction of a particular forecaster is either contributed or withheld. The marginal contribution of forecaster i towards the accuracy of the collective forecast can be given by the equation:

$$MC_i = B(X_a,G(X_c))-B(X_a,G(X_{ci}))$$
$$= L(X_a,G(X_{ci}))-L(X_a,G(X_c)) \quad (3)$$

The marginal contribution for a particular forecaster in a particular instance might well be positive, zero, or negative, depending on whether X_i moves the collective forecast towards or away from X_a.

Hence, the VMP incentive method uses the following type of pay schedule for a forecaster:

$$P_i(X_i,X_{ci},X_v) = F + kL(X_v,G(X_{ci})) - kL(X_v,G(X_i,X_{ci})), \quad (4)$$

where F is a fixed payment, k is a fixed positive coefficient, and X_v is a criterion value (either X_a or a proxy) for judging the accuracy of forecaster predictions. The payment schedule in (4) is simply a linear (affine) transformation of the VMP formula in equation (3), assuming X_v=X_a.

III. Aggregating Predictions Using Precision Weights

This section sets up a base-case example which assumes that X_a is observable at some point in the future, so as to serve as a possible basis for forecaster compensation. Section IV modifies this example by assuming that X_a is either not observable, or is not observed soon enough to serve as a practical basis for forecaster compensation. For this section, assume that forecasters are risk neutral, that the criterion value (X_v) is observable and is set equal to X_a, that all random variables are normally distributed, and that the loss function takes the quadratic form:

$$L(X_a,G(X_c)) = h(X_a-G(X_c))^2, \quad h>0 \tag{5}$$

Since the loss function is quadratic, the optimal prediction is the expected value of X_a. We set h=1, since it makes no qualitative difference to the results.

Suppose further that X is the sum of a constant, α, and two random variables: a signal, S, and an unobserved component, ϵ_a. Each forecaster perceives an I_i, which is the sum of S and a forecaster-specific error term, ϵ_i. The variables S, ϵ_a, and each ϵ_i are independently distributed. The variables are defined or distributed as follows:

$$\begin{aligned} X_a &= \alpha + S + \epsilon_a \\ I_i &= S + \epsilon_i \\ S &\sim N(0,\sigma_s^2) \\ \epsilon_a &\sim N(0,\sigma_a^2) \\ \epsilon_i &\sim N(0,1/\tau_i) \end{aligned} \tag{6}$$

Perhaps due to differences in opportunity, effort, or skill, the expected precision (τ_i) of each forecaster may well be different. We assume that each forecaster i observes I_i and τ_i as private information, observes α and σ_s^2 as common information, and does not observe σ_a^2, ϵ_a, S, X_a, or any ϵ_i. The forecast requisitioner does not observe the private information, and is not assumed to observe the common information either.[4] Instead, suppose that each participating forecaster must submit a prediction, X_i, and a claimed precision, T_i, knowing that the forecast requisitioner will aggregate predictions using the following simple formula:

$$G(X_c) = \sum_{i=1}^{N} T_i X_i \,/\, T_c, \tag{7}$$

where $T_c = \sum_{i=1}^{N} T_i$.

The requisitioner simply takes a weighted average of each prediction X_i, based on the submitted precision weights, T_i, of each forecaster. The optimal $G(X_c)$ is computed as follows:[5]

$$G^* = \alpha + \sum_{i=1}^{N} \tau_i ßI_i \,/\, \tau_c, \tag{8}$$

where $\tau_c = \sum_{i=1}^{N} \tau_i$ and $ß = \sigma_s^2/(\sigma_s^2+1/\tau_c)$.

Given the aggregator function in (7), it is sufficient for unbiasedness that $T_i=\tau_i$ and $X_i=\alpha+ßI_i$ for all forecasters. Note that the optimal X_i depends on τ_c. Since τ_c is not known in advance by each forecaster (though each forecaster may have a fair idea of the likely range), a truthful forecaster would prefer to make his forecast conditional on T_c. Hence, let each forecaster submit a conditional prediction function, $X_i(T_c)$, as well as an unconditional precision weight, T_i.

Suppose now that the forecast requisitioner provides the following definitions:

$$T_{ci} = \sum_{j \# i} T_j \tag{9}$$

$$G(X_{ci}) = \sum_{j \# i} T_j X_j \,/\, T_{ci}$$

and sets up the following pay schedule:[6]

$$P_i(T_i, X_i(T_i), T_{ci}, X_{ci}(T_{ci}), X_a) \tag{10}$$
$$= k(X_a - G(X_{ci}))^2 - k(X_a - G(X_c))^2, \text{ where } k>0$$

In Lundgren (1994), the following proposition is shown to be true [Proof reproduced in Appendix B]:[7]

Proposition 1: If the random variables X_a and I_i are specified as in (6), the forecasts are aggregated according to (7) and (9), and forecasters wish to maximize their expected payoff, where this payoff is given (for any k>0) by (10), then it is a Nash equilibrium for each forecaster to report a truthful precision value ($T_i = \tau_i$) and a conditional prediction function, $X_i(T)$. Further, this vector of reports, when aggregated according to (7), will minimize the expected value of the loss function given in (5). Also, if k=1 in the pay schedule in (10), then, given the number and precision levels of the other forecasters, each forecaster exerts the socially optimal level of effort.

IV. Contemporaneous Criterion Estimates

When actual future measurements of a variable value cannot be used as a criterion value for assessing the accuracy of forecasters' predictions, then it becomes necessary to use an estimated value as the criterion value. Such an estimated value can be termed a "criterion estimate." There are at least two ways of constructing such a criterion estimate. The first way is to base the criterion estimate on the contemporaneous predictions of other forecasters who have issued their predictions simultaneously with the forecaster whose compensation is being determined. The second way is to base the criterion estimate on predictions issued in the future by forecasters predicting the same or similar variable. In this paper we discuss only the contemporaneous criterion estimate technique, not any future criterion estimate technique.

Suppose n forecasters are assigned to provide a forecast with respect to a variable whose precise value will never be observed. An observed value of the variable can therefore never be used to judge the accuracy of forecasters' predictions.

Suppose further that the n forecasters are divided into two, mutually exclusive groups, group A and group B.[8] The n forecasters issue predictions simultaneously, and without foreknowledge of each others' predictions. Hence, the predictions in group B can be used to determine the compensation of forecasters in group A, and the predictions in group A can be used to determine the compensation of forecasters in group B. It might be logical to suppose that the criterion value for forecasters in group A should simply be the collective prediction of the forecasters in group B (and vice versa). However, this approach will not, in general, provide appropriate incentives for the two groups of forecasters to provide appropriate forecasts.

For example, suppose (as in the previous section) that $\alpha+S$ is the humanly estimable component of the random variable X, where α is constant and S is normally distributed with mean zero. If the loss function is quadratic, it is desirable that the criterion value, X_v, fulfill the following condition to assure unbiasedness of the forecasting incentives: $E(X_v|S) = \alpha+S$.

Group B's collective prediction, G_B, is not a suitable criterion estimate for incentive purposes because $E(G_B|S) = E(\alpha+ß_BI_B|S) = \alpha+ß_BS$, where $ß_B=\sigma_s^2/(\sigma_s^2+1/\tau_B)$ and τ_B is the precision of group B's forecast. Since I_B is only a noisy observation of S, $ß_B<1$. Even though $G_B=E(X_a|I_B)$, it is not true that $E(G_B|S)=E(X_a|S)$. Hence, G_B is not a suitable criterion value for incentive purposes in the VMP incentive scheme, even though it is an optimal aggregator of I_B for predictive purposes. Only in the case where τ_B is very large, so that $ß_B\approx1$, would G_B be a suitable criterion estimate.

Ideally, we want a criterion estimate, X_{Be}, from group B such that $E(X_{Be}|S) = \alpha+S$. This can be accomplished if we substitute $T_B=\infty$ into the prediction functions submitted by forecasters in group B. If $T_B=\infty$, then $ß_B = \sigma_s^2/(\sigma_s^2+1/T_B) = \sigma_s^2/(\sigma_s^2+1/\infty) = 1$. Hence, $E(X_{Be}|S) = \alpha+ß_BS = \alpha+S = X$. Since each forecaster must submit a prediction function, not a single prediction, this technique can be used to motivate unbiased predictions of X.

To use this technique, let each forecaster submit a precision weight, T_i, and a prediction function, $X_i(T)$, where T is a value which the forecast requisitioner will later plug into the submitted function. If the forecast requisitioner wishes to compute a criterion estimate (for incentive purposes only), T is set equal to infinity within the forecasters' submitted prediction functions. If the forecast requisitioner wishes to compute a collective prediction or secondary collective prediction, T is set equal to the sum of issued T_i's for the particular group of forecasters from which the collective prediction or secondary collective prediction is sought.

For example, if the method of aggregating predictions being used by the forecast requisitioner is that of taking arithmetic means, then the criterion estimates for groups A and B would be as follows:

$$X_{Ae} = \Sigma\ X_j(T=\infty)T_j/T_A,\ j \in A, \text{ where } T_A = \Sigma\ T_j,\ j \in A, \text{ and} \qquad (11)$$

$$X_{Be} = \Sigma\ X_j(T=\infty)T_j/T_B,\ j \in B, \text{ where } T_B = \Sigma\ T_j,\ j \in B.$$

The collective predictions for groups A and B and for all n forecasters combined (group C) would be as follows:

$$G(X_A) = \Sigma\ X_j(T=T_A)T_j/T_A,\ j \in A, \qquad (12)$$

$$G(X_B) = \Sigma\ X_j(T=T_B)T_j/T_B,\ j \in B, \text{ and}$$

$$G(X_C) = \Sigma\ X_j(T=T_C)T_j/T_C, \text{ all } j, \text{ where } T_C = \Sigma\ T_j, \text{ all } j.$$

For purposes of using the collective forecast (as opposed to calculating the compensation of forecasters), it is best to make use of $G(X_C)$ as the "official" collective forecast, since it is this forecast which incorporates the information of all the forecasters.

The secondary collective predictions for groups A and B would be as follows:

$$\text{If } i \in A, \text{ then } G(X_{Ai}) = \Sigma\ X_j(T=T_{Ai})T_j/T_{Ai},\ j \in A,\ j \# i, \qquad (13)$$

$$\text{where } T_{Ai} = \Sigma\ T_j,\ j \in A,\ j \# i, \text{ and}$$

$$\text{If } i \in B, \text{ then } G(X_{Bi}) = \Sigma\ X_j(T=T_{Bi})T_j/T_{Bi},\ j \in B,\ j \# i,$$

$$\text{where } T_{Bi} = \Sigma\ T_j,\ j \in B,\ j \# i.$$

The pay schedule for any forecaster i in group A can be computed as follows:

$$P_i(T_i, X_i(T), T_{Ai}, X_{Ai}(T), X_{Be}) = F + kL(X_{Be}, G(X_{Ai})) - kL(X_{Be}, G(X_A)), \quad \text{where } k>0. \tag{14}$$

Similarly, the pay schedule for any forecaster i in group B can be computed as follows:

$$P_i(T_i, X_i(T), T_{Bi}, X_{Bi}(T), X_{Ae}) = F + kL(X_{Ae}, G(X_{Bi})) - kL(X_{Ae}, G(X_B)), \quad \text{where } k>0. \tag{15}$$

When forecasters are risk averse, variation in forecaster compensation can be reduced by applying this technique several times using several different groupings of forecasters. An average of the compensation computed under each calculation can then be used to calculate the actual compensation to a particular forecaster. For example, if there are ten forecasters, the nine forecasters other than i can be grouped in as many as 510 (2^9-2) different ways.[9] Computing an average of compensation reduces the variance of compensation, and is therefore beneficial in reducing the risk premia which forecasters would demand in order to enter the forecasting task.[10]

V. Truth-Telling and Multiple Equilibria

With regard to the above incentive scheme, the following two propositions can be stated:

Proposition 2: If the random variables X_a and I_i are given by (6), the forecaster reports are aggregated according to (11), (12), and (13), and individual forecasters wish to maximize the expected value of their payoffs, where these payoffs are given (for any $k>0$) in (14) and (15), then multiple Nash equilibrium outcomes are possible for the reports of precision weights and conditional prediction functions. These equilibrium weights and conditional prediction functions take the form $T_i=K_t\tau_i$ and $X_i(T)=K_\alpha+\beta_B K_I I_i$ for all i, where $\beta_B = K_s\sigma_s^2/(K_s\sigma_s^2+1/T)$, provided $K_s=1/K_t$. The resulting collective prediction is $G(X_C) = \Sigma X_j(T=T_C)T_j/T_C = K_\alpha+\beta_C K_I I_C$, where $\beta_C = K_s\sigma_s^2/(K_s\sigma_s^2+1/K_t\tau_C) = \sigma_s^2/(\sigma_s^2+1/\tau_C)$.

Proof: See Appendix A.

Proposition 3: Under the conditions in Proposition 2, it is a Nash equilibrium for a group of risk-neutral forecasters to submit the socially optimal precision weights and conditional prediction functions. These optimal weights and conditional prediction functions are $T_i=\tau_i$ and $X_i(T)=\alpha+\beta_B I_i$, where $\beta_B = \sigma_s^2/(\sigma_s^2+1/T)$. The resulting collective prediction is $G(X_C) = \Sigma X_j(T=T_C)T_j/T_C = \alpha+\beta_C I_C$, where $\beta_C = \sigma_s^2/(\sigma_s^2+1/\tau_C)$.

Proof: Substitute $K_I=K_s=K_t=1$ and $K_\alpha=\alpha$ in Proposition 2.

Proposition 3 says, in effect, that all forecasters telling the truth is a Nash equilibrium under this incentive scheme. Proposition 2 states that there are many Nash equilibria under this incentive scheme, most of which do not involve truth-telling. The collective prediction is affected by the equilibrium choices of K_α and K_I, but not by K_s and K_t. For example, if $K_\alpha=\alpha+10$, then every forecaster adds the value of 10 to what a truthful prediction would have been. As a consequence, the collective prediction is 10 higher than what it should be. It should be noted, however, that an equilibrium where everyone mendaciously utters $K_\alpha=\alpha+10$ (or $K_\alpha=\alpha-10$) is no more profitable than an honest equilibrium where everyone utters $K_\alpha=\alpha$.

What, then, determines whether the truthful Nash equilibrium would arise, rather than the many possible untruthful Nash equilibria? One factor to consider is to determine which equilibrium is likely to serve as a focal point.[11] There is only one truthful Nash equilibrium, but a multiplicity of mendacious Nash equilibria. Therefore, truth-telling is easily made a focal point for equilibrium, but a mendacious outcome is probably not a focal point. Morality also requires truth-telling. Therefore, even if only one forecaster out of many (with some positive probability) moralistically decides only to tell the truth, while all other forecasters maximize only their expected pay, only the truthful equilibrium can still remain as an equilibrium.[12]

Another way to make the truthful equilibrium the only equilibrium would be to base the criterion value, in part, on a future observation or estimate of the variable being predicted, in addition to the peer-group estimation. This alternative would work, provided that the future observation or estimate is known to be unbiased, even if it is lacking in precision. This alternative method will not be examined further in this paper.

VI. Honesty Versus Ideological Extremism

Truth-telling is not the only possible equilibrium, if, in addition to honest forecasters, we assume the existence of non-optimizing "extremists" who choose to expound one-sided "ideological" positions rather than maximize their expected pay under the forecasting scheme. For example, suppose there exists one truthful forecaster who always issues $K_\alpha=\alpha$ and one untruthful forecaster who always issues $K_\alpha=\alpha+10$, while all other forecasters attempt to maximize expected pay. If we assume that both non-optimizing forecasters issue forecasts with the same level of precision, their average forecast is $K_\alpha=\alpha+5$, so it is a Nash equilibrium for all other forecasters to issue $K_\alpha=\alpha+5$ as well.

The mere existence of "ideological" forecasters will not necessarily lead to biased predictions. For example, suppose there are two "extremists" with opposited biases. One extremist submits $K_\alpha=\alpha+10$ and the other extremist submits $K_\alpha=\alpha-10$. Their average forecast is $K_\alpha=\alpha$, so it is a Nash equilibrium for the optimizing forecasters to submit the truthful report, $K_\alpha=\alpha$. Unless the optimizing forecasters have reason to believe that the ideological forecasters will be biased in a particular direction, truth-telling may still emerge as the equilibrium for optimizing forecasters. Since ideological extremists of one stripe (e.g., pro-environment, anti-business) tend to beget extremists of the opposite stripe (e.g., pro-business, anti-environment), a balance of extremes among non-optimizing forecasters may not be a bad starting assumption for the optimizing forecasters.

Even where ideological positions are known to be unbalanced in a particular direction, there are still reasons to suppose that the incentives will lead forecasters towards truth-telling. For example, suppose there are three non-optimizing forecasters. Forecaster A, a wild extremist, asserts $K_{\alpha}=\alpha+20$. Forecaster B, a mild extremist, asserts $K_{\alpha}=\alpha-5$. Forecaster C, an honest hero, asserts $K_{\alpha}=\alpha$. Since the average among non-optimizers is $K_{\alpha}=\alpha+5$, the remaining forecasters, assumed to be selfish optimizers, mendaciously assert $K_{\alpha}=\alpha+5$. The outcome is mildly biased towards the views of the wild extremist.

The non-optimizing forecasters pay a price (opportunity cost for expected compensation either lost or not gained) for their refusal to optimize. Under a pay schedule with quadratic loss functions, this price is proportional to the square of the deviation from optimal behavior. In the example above, selfish optimizers assert $K_{\alpha}=\alpha+5$. The honest hero who asserts $K_{\alpha}=\alpha$ deviates by 5 and pays a price proportional to 5^2 (say \$25). The mild extremist who asserts $K_{\alpha}=\alpha-5$ deviates by 10 and must pay a price proportional to 10^2 (say \$100). The wild extremist who asserts $K_{\alpha}=\alpha+20$ deviates by 15 and must pay a price proportional to 15^2 (say \$225).

Of the three non-optimizing forecasters, it is clear that the honest hero is under the least financial pressure to alter his behavior. The mild extremist is under significantly greater pressure to alter his behavior, and the wild extremist is under the greatest pressure to alter his behavior. The extremists are under the most financial pressure to alter their behavior, perhaps by moderating their extremist predictions, perhaps by reducing the submitted precision weights on their extreme predictions, or perhaps by dropping out of the incentive scheme altogether. Each of these behavioral responses to the financial pressure induced by the incentive scheme will tend to reduce or eliminate the influence of ideological positions upon the resulting collective forecast, and also reduce the mild financial pressure on honest forecasters to forecast dishonestly.

Indeed, one could look at this financial pressure in reverse, and make the opposite critique. Rather than assume that $K_\alpha=\alpha$ represents "truth," someone might claim that $K_\alpha=\alpha$ simply represents an incorrect consensus based on some conventional mode of thinking. Hence, those who appear to hold extremist views are more likely to have discovered "truth" by means of unconventional thinking, and should not be penalized by financial pressures to compromise their views. One could, of course, question the presupposition of this critique that "conventional" predictions are normally wrong, and that "unconventional" predictions are normally right. Regardless, it should be noted that the "consensus" towards which the financial pressures converge is likely to incorporate, to some extent, the "unconventional" views of those who honestly and conscientiously disagree with the previously "conventional" views.

It should also be noted that merely achieving consensus is not what the VMP incentive scheme rewards. If it is common knowledge that $K_\alpha=\alpha$ is the consensus, then there is no <u>marginal</u> value to a forecast which simply predicts $X=\alpha$. Hence, the forecaster who does no work, except to determine that the consensus K_α is α, achieves zero VMP and zero precision in the estimation of any nonconsensus portion of the forecast. It is only by working on the dissensus portion of the forecast, by attempting to observe the hidden value of the signal S (which is not yet common knowledge), that any forecaster can expect to achieve positive expected VMP and positive precision for the incremental information (I_i) embodied in his forecast.

There is an initial tendency to perceive this potential for mendacious equilibria and/or consensus forecasting as "flaws" in the incentive scheme. Indeed, the outcome is clearly "flawed" if we make comparisons with an unattainable ideal, where every human is always honest regardless of incentive or ideology. Flaws in the human condition abound, but the incentive scheme itself is not flawed, unless a critic of the incentive scheme can point out an alternative scheme which has superior properties when applied to human forecasters.

An alternative scheme which most readily comes to mind is to pay forecasters a fixed remuneration, regardless of forecast. Since the compensation is not contingent upon the accuracy or inaccuracy of the forecast, the forecaster has no incentive to lie, but neither has he an incentive to tell the truth. The forecaster has no financial incentive to exert effort to conduct analysis or to acquire information, nor is there a financial incentive to refrain from issuing forecasts based on ideological bias. It seems likely that the outcome of a fixed salary scheme would be worse than under the incentive scheme outlined here.[13]

VII. Expected Pay of Forecasters

Assuming that truth-telling would emerge as the equilibrium, the incentive scheme just described performs well in eliciting accurate information and in fully utilizing the information available to forecasters. However, the social welfare problem with regard to the optimal use of forecasters requires that there be efficiency on two additional margins: Efficiency on the intensive margin requires that there be optimal levels of effort exerted by each forecaster. Efficiency on the extensive margin requires that an optimal number of forecasters be attracted to the forecasting task. An analysis of the incentives allows us to determine how the incentive scheme can be adjusted to accomplish these two optimizations.

For any given level of precision which a forecaster might provide, there is a cost of exerting the necessary time and effort. Express this relationship as $C(\tau_i)$. On the intensive margin, the social welfare problem (holding constant for the number and type of other forecasters) requires that a forecaster set forth the following amount of effort:

$$\text{maximize } SW(\tau_i) = -L(\tau_{Ci},\tau_i) - C(\tau_i) - C(\tau_{Ci}) \tag{16}$$

This has solution:

$$\partial SW/\partial\tau_i = -L'(\tau_i) - C'(\tau_i) = 0 \tag{17}$$

On the margin, the social benefit from greater forecast accuracy must be balanced against the cost of achieving such accuracy. To compute $L'(\tau_i)$, we must first compute E(L). Breaking down the variables in (5) into their component parts, we have:

$$L = [S+\epsilon_a - \beta_c(S+\epsilon_c)]^2 \qquad (18)$$
$$= [(1-\beta_c)S + \epsilon_a - \beta_c\epsilon_c]^2$$

Taking expectations:

$$E(L) = \sigma_s^2/(\sigma_s^2\tau_c+1)^2 + \sigma_a^2 + \sigma_s^4\tau_c/(\sigma_s^2\tau_c+1)^2 \qquad (19)$$
$$= \sigma_s^2/(\sigma_s^2\tau_c+1) + \sigma_a^2$$

Hence, substituting into (17) we derive:

$$C'(\tau_i) = -L'(\tau_i) = \sigma_s^4/(\sigma_s^2\tau_c+1)^2 \qquad (20)$$

Equation (30) describes the ideal level of incentive for inducing optimal levels of effort from forecasters. We now check whether the incentive scheme provides this desired level of incentive. Under the incentive scheme in (14):

$$E(P_i) = kE(X_{Be}-G(X_{Ai}))^2 - kE(X_{Be}-G(X_A))^2 \qquad (21)$$
$$= kE\{ S\tau_{Ai}/(\sigma_s^2+1/\tau_{Ai}) + \epsilon_B - \epsilon_{Ai}\sigma_s^2/(\sigma_s^2+1/\tau_{Ai}) \}^2$$
$$- kE\{ S\tau_A/(\sigma_s^2+1/\tau_A) + \epsilon_B - \epsilon_A\sigma_s^2/(\sigma_s^2+1/\tau_A) \}^2$$
$$= k\sigma_s^2\{\tau_{Ai}/(\sigma_s^2+1/\tau_{Ai})\}^2 + k(1/\tau_{Ai})\{\sigma_s^2/(\sigma_s^2+1/\tau_{Ai})\}^2$$
$$- k\sigma_s^2\{\tau_A/(\sigma_s^2+1/\tau_A)\}^2 - k(1/\tau_A)\{\sigma_s^2/(\sigma_s^2+1/\tau_A)\}^2$$
$$= k\sigma_s^4\tau_i/\{(\sigma_s^2\tau_{Ai}+1)(\sigma_s^2\tau_A+1)\}$$

$$\partial E(P_i)/\partial\tau_i = k\sigma_s^4/(\sigma_s^2\tau_A+1)^2 \qquad (22)$$

A comparison of (20) and (22) indicates that k<1 in the pay equation is required if the incentive level under the payment scheme is to be set equal to the optimal incentive level on the intensive margin. This optimal value of k is the ratio of the two quantities, as follows:

$$k^* = (\sigma_s^2\tau_A+1)^2/(\sigma_s^2\tau_c+1)^2 \qquad (23)$$

On the extensive margin, the social welfare problem (holding constant for the number and type of other forecasters) requires that a forecaster be added to the forecasting task only if the incremental benefit to society exceeds the cost of the forecaster's total effort. An <u>approximation</u> to the total level of incentive required for a forecaster on the extensive margin is given as follows:

$$
\begin{aligned}
& E\,\{L(G(X_{ci}),X_a) - L(G(X_c),X_a)\} \qquad (24)\\
&= E(X_a-G(X_{ci}))^2 - E(X_a-G(X_c))^2\\
&= E\{\ S\tau_{ci}/(\sigma_s^2+1/\tau_{ci}) + \epsilon_a - \epsilon_{ci}\sigma_s^2/(\sigma_s^2+1/\tau_{ci})\ \}^2\\
&\quad - E\{\ S\tau_c/(\sigma_s^2+1/\tau_c) + \epsilon_a - \epsilon_c\sigma_s^2/(\sigma_s^2+1/\tau_c)\ \}^2\\
&= \sigma_s^2\{\tau_{ci}/(\sigma_s^2+1/\tau_{ci})\}^2 + \sigma_a^2 + (1/\tau_{ci})\{\sigma_s^2/(\sigma_s^2+1/\tau_{ci})\}^2\\
&\quad - \sigma_s^2\{\tau_c/(\sigma_s^2+1/\tau_c)\}^2 - \sigma_a^2 - (1/\tau_c)\{\sigma_s^2/(\sigma_s^2+1/\tau_c)\}^2\\
&= \sigma_s^4\tau_i/\{(\sigma_s^2\tau_{ci}+1)(\sigma_s^2\tau_c+1)\}
\end{aligned}
$$

The formula is only an approximation because it takes no account of possible changes in forecaster effort as a result of the reduction in total number of forecasters.[14] If we assume F=0 in the pay schedule, then the required level of k in the pay schedule needed to induce efficiency on the extensive margin is approximately the ratio of (21) and (24):

$$k^* = \{(\sigma_s^2\tau_A+1)(\sigma_s^2\tau_{Ai}+1)\} \,/\, \{(\sigma_s^2\tau_c+1)(\sigma_s^2\tau_{ci}+1)\} \qquad (25)$$

If the number of forecasters is sufficiently large that $\tau_{Ai}/\tau_A \approx 1$ and $\tau_{ci}/\tau_c \approx 1$, then k* in (25) is approximately the same as k* in (23), so that even with F=0, it is possible to adjust the incentive scheme to have approximate efficiency on both the intensive and extensive margins. If there are only a few forecasters, it may be more advantageous to adjust both F and k so as to obtain efficiency on both margins. This would imply setting k for the intensive margin as in (23) and adjusting F for the given k* so as to obtain the right incentive on the extensive margin. This requires F<0, meaning that forecasters receive a lump-sum reduction from their remuneration.

VIII. Conclusion

An incentive scheme to elicit predictions of unobservable variable values can be constructed by using the predictions of other forecasters as a criterion estimate for judging the accuracy of forecaster predictions. Provided that this criterion estimate is appropriately constructed, a group of forecasters issuing predictions simultaneously can be mutually motivated to provide truthful predictions as a Nash equilibrium. Untruthful equilibria can exist, but there are reasons to suppose that the truthful equilibrium is more likely to emerge. The incentive scheme can be adjusted so as to provide appropriate incentives on the intensive margin to induce optimal effort levels and on the extensive margin to induce entry of near-optimal numbers of forecasters for the forecasting task.

The author is unaware of any alternative incentive scheme for eliciting unobservable variable values, let alone an incentive scheme that would have better properties. The only basis for judging the desirability of the incentive scheme is to compare it with a non-incentive scheme of forecasting (e.g., eliciting forecasts from salaried employees). Forecasts resulting from a non-incentive scheme are probably less reliable than forecasts resulting from an incentive scheme that has truth-telling and incentives for effort as one of its Nash equilibrium outcomes. Therefore, if forecasts of unobservable variables must be elicited, it seems more preferable than not to use the incentive scheme described herein.

APPENDIX A. PROOF OF PROPOSITION 2.

To determine whether this is a Nash equilibrium, first assume that all other forecasters submit $T_i=K_t\tau_i$ and $X_i(T)=K_\alpha+\beta_B K_I I_i$ for all i, where $\beta_B=K_s\sigma_s^2/(K_s\sigma_s^2+1/T)$ and $K_s=1/K_t$. Then determine whether it is optimal for a particular forecaster to abide by the same strategy. Define:

$$I_A = \sum_{i\in A} \tau_i I_i \,/\, \tau_A$$

$$\tau_{Ai} = \tau_A - \tau_i$$

$$I_{Ai} = \sum_{j\in A-i} \tau_j I_j \,/\, \tau_{ci} \tag{A.1}$$

$$\epsilon_A = I_A - S$$

$$\epsilon_{Ai} = I_{Ai} - S$$

$$I_B = \sum_{i\in B} \tau_i I_i \,/\, \tau_B$$

$$\epsilon_B = I_B - S$$

The pay schedule in (21) is equivalent to:

$$\begin{aligned} &P_i(T_i,X_i(T_i),T_{Ai},X_{Ai}(T_{Ai}),X_{Be}) \\ &\quad = (X_{Be}-G(X_{Ai}))^2 - (X_{Be}-G(X_{Ai}))^2 \\ &\quad = -2X_{Be}G(X_{Ai}) + G(X_{Ai})^2 + 2X_{Be}G(X_A) - G(X_A)^2 \end{aligned} \tag{A.2}$$

Of the three random variables, X_{Be}, $G(X_{Ai})$, and $G(X_A)$, the individual forecaster only has control over $G(X_A)$. Hence, risk-neutral forecaster i maximizes the expectation of the following quantity:

$$M = 2X_{Be}G(X_A) - G(X_A)^2 \tag{A.3}$$

Under the assumption that all other forecasters play the Nash equilibrium strategy, X_{Be} and $G(X_A)$ take on the following values:

$$X_{Be} = K_\alpha + K_I(S+\epsilon_B) \qquad (A.4)$$

$$G(X_A) = (K_\alpha K_t\tau_{Ai} + T_i\alpha_i)/T_A$$

$$+ (S+\epsilon_{Ai})K_s\sigma_s^2K_IK_t\tau_{Ai}/(K_s\sigma_s^2T_A+1)$$

$$+ (S+\epsilon_i)K_iT_iS_i/(S_iT_A+1)$$

where $T_A=K_t\tau_{Ai}+T_i$. It is assumed in (A.4) that forecaster i chooses the conditional prediction function based on choosing three parameters (α_i,K_i,S_i) for the formula $X_i(T)=\alpha_i+\beta_iK_iI_i$, where $\beta_i=S_i/(S_i+1/T)$. These three parameters plus T_i are the choice variables.

Substituting (A.4) into (A.3) and taking expectations of the random variables, we obtain the following values for the terms in M:

$$2X_{Be}G(X_A) = 2K_\alpha(K_\alpha K_t\tau_{Ai} + T_i\alpha_i)/T_A \qquad (A.5)$$

$$+ 2K_I^2\sigma_s^4K_sK_t\tau_{Ai}/(K_s\sigma_s^2T_A+1)$$

$$+ 2K_I\sigma_s^2K_iT_iS_i/(S_iT_A+1)$$

$$-G(X_A)^2 = -(K_\alpha^2K_t^2\tau_{Ai}^2 + 2K_\alpha K_t\tau_{Ai}T_i\alpha_i + T_i^2\alpha_i^2)/T_A^2$$

$$-(\sigma_s^2+1/\tau_{Ai})K_s^2\sigma_s^4K_I^2K_t^2\tau_{Ai}^2/(K_s\sigma_s^2T_A)^2$$

$$-(\sigma_s^2+1/\tau_i)K_i^2T_i^2S_i^2/(S_iT_A+1)^2$$

$$-2\sigma_s^4K_sK_IK_t\tau_{Ai}K_iT_iS_i/[(K_s\sigma_s^2T_A+1)(S_iT_A+1)]$$

We next take derivatives of the terms in M to obtain the first-order conditions for maximization of $E(P_i)$:

$$\partial M/\partial\alpha_i = 2K_\alpha T_i/T_A - 2T_i^2\alpha_i/T_A^2 - 2K_\alpha K_t\tau_{Ai}T_i/T_A^2 = 0 \qquad (A.6)$$

$$\partial M/\partial K_i = 2K_I\sigma_s^2T_iS_i/(S_iT_A+1) \qquad (A.7)$$

$$- 2(\sigma_s^2+1/\tau_i)K_iT_i^2S_i^2/(S_iT_A+1)^2$$

$$- 2\sigma_s^4K_sK_IK_t\tau_{Ai}T_iS_i/[(K_s\sigma_s^2T_A+1)(S_iT_A+1)] = 0$$

$$\partial M/\partial S_i = 2K_I\sigma_s^2K_iT_i/(S_iT_A+1) \tag{A.8}$$

$$- 2(K_I\sigma_s^2K_iT_iS_iT_A/(S_iT_A+1)^2$$

$$- 2(\sigma_s^2+1/\tau_i)K_i^2T_i^2S_i/(S_iT_A+1)^2$$

$$+ 2(\sigma_s^2+1/\tau_i)K_i^2T_i^2S_i^2T_A/(S_iT_A+1)^3$$

$$- 2\sigma_s^4K_sK_IK_t\tau_{Ai}K_iT_i/[(K_s\sigma_s^2T_A+1)(S_iT_A+1)]$$

$$+ 2\sigma_s^4K_sK_IK_t\tau_{Ai}K_iT_iS_iT_A/[(K_s\sigma_s^2T_A+1)(S_iT_A+1)^2] = 0$$

$$\partial M/\partial T_i = 2K_\alpha\alpha_i/T_A - 2K_\alpha(K_\alpha K_t\tau_{Ai} + T_i\alpha_i)/T_A \tag{A.9}$$

$$+ 2K_\alpha^2K_t^2\tau_{Ai}^2/T_A^3 - 2T_i\alpha_i^2/T_A^2 + 2T_i^2\alpha_i^2/T_A^3$$

$$- 2K_\alpha K_t\tau_{Ai}\alpha_i/T_A^2 + 4K_\alpha K_t\tau_{Ai}T_i\alpha_i/T_A^3$$

$$- 2K_I^2\sigma_s^6K_s^2K_t\tau_{Ai}/(K_s\sigma_s^2T_A+1)^2$$

$$+ 2K_I\sigma_s^2K_iS_i/(S_iT_A+1) - 2K_I\sigma_s^2K_iT_iS_i^2/(S_iT_A+1)^2$$

$$+ 2(\sigma_s^2+1/\tau_{Ai})K_s^3\sigma_s^6K_I^2K_t^2\tau_{Ai}^2/(K_s\sigma_s^2T_A+1)^3$$

$$- 2(\sigma_s^2+1/\tau_i)K_i^2T_iS_i^2/(S_iT_A+1)^2$$

$$+ 2(\sigma_s^2+1/\tau_i)K_i^2T_i^2S_i^3/(S_iT_A+1)^3$$

$$- 2\sigma_s^4K_sK_IK_t\tau_{Ai}K_iS_i/[(K_s\sigma_s^2T_A+1)(S_iT_A+1)]$$

$$+ 2\sigma_s^6K_s^2K_IK_t\tau_{Ai}K_iT_iS_i/[(K_s\sigma_s^2T_A+1)^2(S_iT_A+1)]$$

$$+ 2\sigma_s^4K_sK_IK_t\tau_{Ai}K_iT_iS_i^2/[(K_s\sigma_s^2T_A+1)(S_iT_A+1)^2] = 0$$

Finally, we substitute $\alpha_i=K_\alpha$, $S_i=K_s\sigma_s^2$, $T_i=K_t\tau_i$, $K_i=K_I$, and $T_A=K_t\tau_A$ into (A.6)-(A.9) to see if it is optimal for forecaster i to choose this strategy if all other forecasters choose the same strategy. Equation (A.6) and the α-terms in (A.9) zero out as expected. Equations (A.7), (A.8), and the non-α terms in (A.9) zero out only if $K_sK_t=1$, which requires $K_s=1/K_t$. Hence, any strategy combination of the form indicated in Proposition 2 is a Nash equilibrium.

APPENDIX B. PROOF OF PROPOSITION 1.

Proof of first two sentences: To determine whether this can be a Nash equilibrium, suppose all other forecasters submit $T_i=\tau_i$ and $X_i(T_c)=\beta_c I_i$ where $\beta_c=\sigma_s^2/(\sigma_s^2+1/T_c)$. We then ask whether it is optimal for a particular forecaster to abide by the same strategy. Define:

$$
\begin{aligned}
I_c &= \sum_{i=1}^{N} \tau_i I_i / \tau_c \\
\tau_{ci} &= \tau_c - \tau_i \\
I_{ci} &= \sum_{j \# i} \tau_j I_j / \tau_{ci} \qquad \text{(B.1)} \\
\epsilon_c &= I_c - S \\
\epsilon_{ci} &= I_{ci} - S
\end{aligned}
$$

If we break down the variables in (10) into their component parts we obtain:

$$
\begin{aligned}
&P_i(T_i,X_i,...) \qquad \text{(B.2)} \\
&= 2(S+\epsilon_a)\{X_iT_i/(T_i+\tau_{ci})+\beta_c(S+\epsilon_{ci})\tau_{ci}/(T_i+\tau_{ci})-\beta_{ci}(S+\epsilon_{ci})\} \\
&+ \beta_{ci}^2(S+\epsilon_{ci})^2 - \{X_iT_i/(T_i+\tau_{ci})+\beta_c(S+\epsilon_{ci})\tau_{ci}/(T_i+\tau_{ci})\}^2
\end{aligned}
$$

where $\beta_c = \sigma_s^2/(\sigma_s^2+1/(T_i+\tau_{ci}))$

and $\beta_{ci} = \sigma_s^2/(\sigma_s^2+1/\tau_{ci})$

Taking expectations we obtain:

$$
\begin{aligned}
E(P_i) = {} & 2\beta_i I_i\{X_iT_i/(T_i+\tau_{ci})+\beta_c\beta_i I_i\tau_{ci}/(T_i+\tau_{ci})-\beta_{ci}\beta_i I_i\} \qquad \text{(B.3)} \\
& + 2(\beta_i/\tau_i)\{\beta_c\tau_{ci}/(T_i+\tau_{ci})-\beta_{ci}\} + \beta_{ci}^2\{\beta_i^2 I_i^2+\beta_i/\tau_i+1/\tau_{ci}\} \\
& - X_i^2T_i^2/(T_i+\tau_{ci})^2 - 2X_i\beta_c\beta_i I_i T_i\tau_{ci}/(T_i+\tau_{ci})^2 \\
& - \beta_c^2\{\beta_i^2 I_i^2+\beta_i/\tau_i+1/\tau_{ci}\}\tau_{ci}^2/(T_i+\tau_{ci})^2
\end{aligned}
$$

where $\beta_i = \sigma_s^2/(\sigma_s^2+1/\tau_i)$

First-order conditions for maximization of expected pay require:

$$(\partial P/\partial X_i) = 2ß_i I_i T_i/(T_i+\tau_{ci}) - 2X_i T_i^2/(T_i+\tau_{ci})^2 \tag{B.4}$$
$$- 2ß_c ß_i I_i T_i \tau_{ci}/(T_i+\tau_{ci})^2 = 0$$

$$(\partial P/\partial T_i) = 2ß_i I_i X_i/(T_i+\tau_{ci}) - 2ß_i I_i X_i T_i/(T_i+\tau_{ci})^2 \tag{B.5}$$
$$- 2ß_i^2 I_i^2 ß_c \tau_{ci}/(T_i+\tau_{ci})^2 - 2ß_i \tau_{ci} ß_c/(\tau_i(T_i+\tau_{ci})^2)$$
$$- 2T_i X_i^2/(T_i+\tau_{ci})^2 + 2T_i^2 X_i^2/(T_i+\tau_{ci})^3$$
$$- 2\tau_{ci} X_i ß_c ß_i I_i/(T_i+\tau_{ci})^2 + 4T_i \tau_{ci} X_i ß_c ß_i I_i/(T_i+\tau_{ci})^3$$
$$+ 2\tau_{ci}^2 ß_c^2 (ß_i^2 I_i^2+ß_i/\tau_i+1/\tau_{ci})/(T_i+\tau_{ci})^3$$
$$+ 2ß_i^2 I_i^2 \tau_{ci} ß_c/\{(T_i+\tau_{ci})^2[(T_i+\tau_{ci})\sigma_s^2+1]\}$$
$$+ 2ß_i \tau_{ci} ß_c/\{\tau_i(T_i+\tau_{ci})^2[T_i+\tau_{ci})\sigma_s^2+1]\}$$
$$- 2T_i \tau_{ci} X_i ß_c ß_i I_i/\{T_i+\tau_{ci})^3[(T_i+\tau_{ci})\sigma_s^2+1]\}$$
$$- 2\tau_{ci}^2 ß_c^2 [ß_i^2 I_i^2+ß_i/\tau_i+1/\tau_i]/\{(T_i+\tau_{ci})^3[(T_i+\tau_{ci})\sigma_s^2+1]\}$$
$$= 0$$

It can be verified by substitution that $T_i=\tau_i$ and $X_i=ß_c I_i$ solves (B.4) and (B.5). Hence, it is a Nash equilibrium for all forecasters to submit $T_i=\tau_i$ and $X_i=ßI_i$, which is socially ideal.

<u>Proof of third sentence:</u> If we substitute $T_i=\tau_i$ and $X_i=ß_c I_i$ into (B.3) and take the unconditional expectation, we obtain:

$$E(P) = \sigma_s^4/(\sigma_s^2+1/\tau_c) - \sigma_s^4/(\sigma_s^2+1/\tau_{ci}) \tag{B.6}$$

Since $\tau_c=\tau_i+\tau_{ci}$, E(P) is a function of τ_i. Express this relationship as $P(\tau_i)$. There is also a cost of exerting effort, which results in a given level of precision. Express this relationship as $C(\tau_i)$. The risk-neutral forecaster must solve:

$$\text{maximize } U(\tau_i) = P(\tau_i) - C(\tau_i) \tag{B.7}$$

This has solution:

$$\partial U/\partial \tau_i = P'(\tau_i) - C'(\tau_i) = 0 \tag{B.8}$$

Taking derivatives of (B.6) while taking τ_{ci} as given, we obtain:

$$C'(\tau_i) = P'(\tau_i) = \sigma_s^4/(\sigma_s^2\tau_c+1)^2 \tag{B.9}$$

The social welfare problem (holding constant for the number and type of forecasters) requires that a forecaster set forth the following amount of effort:

$$\text{maximize } SW(\tau_i) = -L(\tau_{ci},\tau_i) - C(\tau_i) - C(\tau_{ci}) \tag{B.10}$$

This has solution:

$$\partial SW/\partial \tau_i = -L'(\tau_i) - C'(\tau_i) = 0 \tag{B.11}$$

To compute $L'(\tau_i)$, we must first compute E(L). Breaking down the variables in (5) into their component parts, we have:

$$\begin{aligned} L &= [S+\epsilon_a - \beta_c(S+\epsilon_c)]^2 \\ &= [(1-\beta_c)S + \epsilon_a - \beta_c\epsilon_c]^2 \end{aligned} \tag{B.12}$$

Taking expectations:

$$\begin{aligned} E(L) &= \sigma_s^2/(\sigma_s^2\tau_c+1)^2 + \sigma_a^2 + \sigma_s^4\tau_c/(\sigma_s^2\tau_c+1)^2 \\ &= \sigma_s^2/(\sigma_s^2\tau_c+1) + \sigma_a^2 \end{aligned} \tag{B.13}$$

Hence, substituting into (B.11) we derive:

$$C'(\tau_i) = -L'(\tau_i) = \sigma_s^4/(\sigma_s^2\tau_c+1)^2 \tag{B.14}$$

Comparison of (B.9) and (B.14) shows that the forecaster always exerts the socially optimal level of effort.

FOOTNOTES

1. NOTICE OF PATENT ISSUED: This paper describes a method of economic incentives, involving plural-forecaster payment systems, upon which the author and inventor has been issued a patent (U.S. Patent 5,608,620). The patent on this invention only restricts actual use of the described invention; it does not restrict in any way the verbal or written discussion, description, or criticism of that invention.

2. See, for example, Samuelson (1959) and many others.

3. It is the primary purpose of this paper to explore the best incentives for eliciting predictions, not the best methods for aggregating predictions. Different prediction problems may call for different methods of aggregation.

4. The "information" in this model refers to all bases for rational forecasts, including both objective data and subjective judgement. Certainly, different humans can interpret the same data quite differently--forecasting is not a purely mechanical/mathematical process.

5. The optimal combination of information is based on standard statistical theory. The derivation is not shown here.

6. The pay schedule is equivalent to assuming F=0 in equation (4).

7. See Propositions 3 and 4 in Lundgren (1994).

8. The criterion estimate technique set forth in this paper will not work so well if any attempt is made to bring about overlapping membership in groups A and B. On the other hand, there is no problem in failing to exhaust use of all solicited and available forecasters between groups A and B, though it may be generally preferable to make use of all the solicited and available forecasts.

9. Assuming forecaster i is in group A, we exclude the grouping in which all other forecasters are part of group A since this prevents computation of X_B. We also exclude the grouping in which all other forecasters are part of group B, since this prevents computation of X_{Ai}. Hence, 2 of the 2^9 possible groupings are not counted.

10. Additional consequences of forecaster risk aversion are discussed in Lundgren (1994).

11. Scherer and Ross (1990, p. 266) explain focal points in the following terms: "In a variety of problems, when behavior must

be coordinated tacitly... there is a tendency for choices to converge on... [a] focal point. The focal points chosen may owe their prominence to analogy, symmetry, precedent, aesthetic considerations, or even the accident of arrangement; but they must in any event have the property of uniqueness." The theory of focal points was originally developed by Schelling (1960), and formally tested by Mehta, et al (1994).

12. Proving uniqueness of equilibrium may be a near impossibility. However, it is clear that the mendacious equilibria pointed out in Proposition 2 cannot occur if one forecaster insists on truthful prediction making.

13. An interesting hybrid of the peer-group incentive scheme and the fixed compensation scheme would be to allow forecasters under the incentive scheme to issue two forecasts, one for incentive purposes and the other independent of compensation. The incentive scheme would induce forecasters to exert effort, while the second set of forecasts would allow forecasters to express their "conscience." One could then analyze whether the two sets of forecasts differed significantly, and whether the non-incentive forecasts perform better or worse.

14. Further discussion of the extensive margin is contained in Lundgren (1994).

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