EFFICIENCY OF THE INFORMATION INCENTIVES OF A FINANCIAL MARKET

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ABSTRACT

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The paper analyzes the efficiency aspects of a futures market with costly information gathering. The model assumes risk-neutral speculators who may choose either to become informed (at a cost) or to remain uninformed. The equilibrium assumes rational expectations on the part of speculators, and consistent conjectures (Nash equilibrium in demand curves, assuming noncollusive, imperfect competition). Computer simulation of the model indicates that speculators have no incentive to become informed unless a) there are a large number of uninformed speculators in the market, or b) there is a substantial amount of noise (at least some noise is necessary). The expected revenues of informed speculators tends to be proportional to the amount of noise.

Computer simulation indicates considerable inefficiency of the futures market compared with an optimal forecasting institution. Most of the deadweight loss from market prediction of future prices occurs because either too many or too few speculators choose to become informed, rather than mainly because noise trading makes the market price prediction less accurate than it otherwise could be. Depending on the extent of noise trading and other factors, speculators can receive either more or less than the value marginal product (VMP) of their contribution to the accuracy of the futures price. When speculators earn more (less) than VMP, too many (too few) speculators are induced to become informed. This, plus the pricing inaccuracy contributed by noise trading, causes the overall inefficiency of the futures market.

JEL CODES: G14 Information and Market Efficiency
D80 Information and Uncertainty
D84 Expectations; Speculations

The purpose of this paper is to analyze a hypothesis advanced by Samuelson (1957) that futures markets tend to over-reward speculators relative to the social value of the information which they contribute to the marketplace. A relevant quote from Samuelson reads as follows:

> "Suppose my reactions are not better than those of other speculators but rather just one second quicker.... Would anyone be foolish enough to argue that in my absence the equilibrium pattern would fail to be reestablished? By hypothesis, my sole contribution is to have it established one second sooner than otherwise. Now even a second counts: and after crops fail, society should even in the first second begin to reduce its consumption of grain. The worth of this one-second's lead time to society is perhaps $5, and if we for the sake of the argument accept a Clarkian naive-productivity theory of ethical deservingness, we might say I truly deserve $5. Actually, however, I get a fortune.... There is no necessary correspondence between the income effects realized by any person's actions and the amount of meritorious substitutions that his actions can alone bring into being." (p. 209)

Samuelson's comment raises two issues: 1) Are financial markets equitable? 2) Are financial markets efficient? The issue of efficency is indirectly implied. If speculators are over-rewarded for their informational efforts, then too many speculators will enter the market and too much resources will be spent collecting information. Contrariwise, if speculators are under-rewarded for their informational efforts, too few speculators will enter the market and too little resources will be spent collecting information. Hence, the question of whether speculators are overcompensated or undercompensated for their efforts has clear efficiency implications.

The issue of equity is raised by Samuelson's sneer at the alleged naivete of J.B. Clark's ethical interpretation of marginal productivity theory. Samuelson presents no argument against Clarkian ethical theory. Rather, Samuelson provisionally adopts Clarkian ethical theory for the sake of argument, to show that financial markets are unjust. Samuelson's uncompleted syllogism might run as follows: (1) Clarkian ethical theory allows for the greatest amount of inequality which is ethically justifiable, but (2) financial market rewards are even more unequal. Therefore, (3) financial market rewards are unjust. Economic philosophers can (and have) disputed the major premise (1), to avoid the conclusion (3).[1] This paper investigates the truth of the minor premise (2).

Samuelson's challenge to the equity and efficiency of financial markets rests mainly on conjecture, rather than a formal proof based on a specific model of the functioning of financial markets. Samuelson's example implicitly makes extreme assumptions about the liquidity of financial markets. Is it really possible to earn a "fortune" by trading large volumes in only one second? Or would it actually take several minutes or even a few hours to trade the necessary volume to earn a fortune? If the latter is the case, it is no longer plausible to claim that the value of the speculator's information to society is only $5.

Finally, an issue which would not be raised in 1957, but which is normally raised today, is whether Samuelson's example is consistent with rational expectations on the part of the other speculators. Does the earning of a "fortune" require that the other speculators be unaware that the informed speculator has superior information? Presumably, if uninformed speculators are aware that informed speculators are trading in the marketplace, their trading strategies might be different from how they might otherwise trade. It is important to know the conditions (if any) under which Samuelson's conjecture actually holds true.

Section I describes the basic model which this paper uses to analyze Samuelson's hypothesis. Section II provides the mathematical detail of this basic model. Section III, in conjunction with Appendices A, B, and C, derives the optimal strategies of speculators, computes the equilibrium equations for any finite number of speculators, and also solves the model under the assumption of costless free entry by uninformed speculators. Section IV calculates the welfare loss of a futures market compared with an optimal forecasting institution. Section V considers alternative definitions of VMP and selects two definitions of VMP as being most useful for analyzing Samuelson's implied equity and efficiency arguments. Section VI analyzes Samuelson's example of the short-term speculator, and shows that very likely the short-term speculator does earn rewards in excess of VMP. Section VII presents the results of computer calculations of welfare losses and reward/VMP ratios for 2,430 cases involving alternative selections of four key parameter values. Section VIII summarizes the key findings and suggests reasons for believing that the conclusions would be valid for alternative models of financial markets.

I. Basic Model

I take it that Samuelson's main point is to assert that "there is no necessary correspondence" between a speculator's income and the VMP of the information which he provides. The example of the quick-witted speculator is solely intended to make this conjecture seem plausible. If the hypothesis has universal validity, then it should be just as true of a model with only one period of speculation, as it would be of a model with several periods of speculation. As it happens, Samuelson's conjecture is true for a model with only one period of speculation. Therefore, extension of the model to several periods is not necessary to prove Samuelson's basic conjecture, even though analysis of the interesting example of the quick-witted speculator might require such extension.

Futures markets can be interpreted as making implicit predictions about future commodity prices. Since inaccurate price predictions can lead to misallocation of resources, financial economists have frequently argued about whether such market predictions exhibit rational expectations.[2] The "efficient markets" hypotheses of financial theory posits that market prices fully reflect all information available to market participants. The terms, "weak-form efficiency," "semistrong-form efficiency," and "strong-form efficiency," simply place different restrictions on the types of information sets which are presumed to be available to market traders, and against which they form rational expectations.

As Grossman and Stiglitz (1980) point out, this view of market "efficiency" is rather limited. If information is costly to collect, a rational expectations equilibrium which fully revealed all information would make it unprofitable for any speculator to acquire information in the first place. Their solution to this paradox is to posit the existence of "noise" which partly obscures the revelation of information, and thereby enables speculators to profit by becoming informed. Consequently, if information is costly, theory does not permit an informed market equilibrium to exhibit fully revealing rational expectations. Additionally, if information is costly, market efficiency requires a proper balancing of social costs and benefits of information acquisition, so that acquiring perfect information can be inefficient, if it costs too much.

The Grossman and Stiglitz (G-S) model assumes perfect competition (Bertrand conjectures) by speculators. As Kyle (1989) points out, this assumption may not be valid when informed traders are sufficiently few and sufficiently large that their trading activity significantly affects market price. Kyle instead assumes imperfect competition (consistent conjectures or Nash equilibrium in speculator demand curves). In the G-S model, as market traders become more risk neutral, they begin to trade so aggressively that it is no longer profitable for speculators to become informed. This oddity of the G-S model is eliminated in the Kyle model.

Both the Kyle model and the G-S model assume that all informational and noise variables are normally distributed. Additionally, they assume that all speculators have negative exponential utility functions with constant absolute risk aversion. Risk neutrality is a special case in the limit as risk aversion approaches zero in these models. The assumption of risk neutrality which I make in this paper simplifies all calculations. Since risk-neutral speculation is incompatible with a perfectly competitive solution for market equilibrium when information is costly, it is necessary to use the imperfectly competitive solution concept.

An advantage of the G-S and Kyle models is that optimal equilibrium speculator strategies can be expressed in linear form. In Theorem 5.1, Kyle (1989) demonstrates that a linear strategy by any speculator dominates all nonlinear strategies if the residual supply curve facing the speculator is also linear, and satisfies certain second-order conditions. This theorem is useful because it allows us to posit linear strategies right from the start, and means that we need only calculate the optimal parameter values for the linear strategies. Kyle (1989) does not prove or disprove the existence of nonlinear equilibria.

The model of this paper is both a simplification and an extension of the Kyle model. It simplifies Kyle's model by assuming risk-neutral speculators, and it extends Kyle's model by assuming that the supply of the speculative asset is upward-sloping. Kyle's model assumes that the supply curve is vertical, which is a special case of the extended model used here. The extension to upward-sloping supply curves is necessary to assure that information collection has at least some social value. If the supply curve were vertical, lack of information would create no deadweight loss, and consequently any positive amount of costly information collection would be socially wasteful.

The model of this paper is intended to be the simplest possible model consistent with investigating Samuelson's conjecture.

II. Assumptions of Basic Model

The basic model assumes two periods. In the first period, the interaction of farmers and market speculators determines the predicted futures price, P_1. In the second period, the normal interaction of supply and demand determines the future spot price, P_2.

Farmers' supply is based on the observed futures price:

$$Q_s = A_s + B_s P_1 \tag{1}$$

Future spot demand is based on the future spot price:

$$Q_d = A_d - B_d P_2 + X, \quad \text{where } X \text{ is random} \tag{2}$$

The period 2 spot market equilibrium is therefore:

$$A_s + B_s P_1 = A_d - B_d P_2 + X \tag{3}$$

Equations (1)-(3) provide the fundamentals. The speculative action occurs in period 1. Speculators are risk neutral, and may be either informed or uninformed. There is also exogenously specified noise trading.

There are M uninformed speculators, indexed by m=1, 2,..., M. Each uninformed speculator has a linear speculative demand of the form:

$$Q_m = A_u - B_u P_1 \tag{4}$$

There are N informed speculators, indexed by n=1, 2,..., N. Each informed speculator observes a different piece of information, I_n. Each informed speculator has a linear speculative demand of the form:

$$Q_n = A_i - B_i P_1 + G_i I_n \tag{5}$$

Noise trader supply is exogenously specified as:

$$Q_z = Z, \quad \text{where } Z \text{ is random} \tag{6}$$

Setting farmers' supply plus noise trader supply equal to the demands of M uninformed speculators and N informed speculators, we obtain the futures market equilibrium:

$$A_s + B_s + Z = MA_u - MB_u P_1 + NA_i - NB_i P_1 + G_i I_i, \tag{7}$$

where

$$I_i = \sum_{n=1}^{N} I_n \tag{8}$$

Rearranging (7) we obtain:

$$P_1 = \frac{A_0 - Z + G_i I_i}{B_0} \tag{9}$$

where $$A_0 = -A_s + MA_u + NA_i \tag{10}$$

and $$B_0 = B_s + MB_u + NB_i \tag{11}$$

Rearranging (3) we obtain:

$$P_2 = \frac{A_{ds} - B_s P_1 + X}{B_d} \quad (12)$$

where $A_{ds} = A_d - A_s$ (13)

From (9) and (12) the equation for the price margin becomes:

$$(P_2 - P_1) = \frac{A_{ds}+X}{B_d} - \frac{B_{sd}[A_0 - Z + G_i I_i]}{B_d B_0} \quad (14)$$

where $B_{sd} = B_s + B_d$ (15)

All random variables are normally distributed with means of zero. X is a composite variable which is the sum of a humanly observable signal, S, and an unobservable random component, E_a. Only the informed speculators make an observation of S. This observation is clouded by an error term, E_n, which is uncorrelated across speculators. The basic random variables are distributed as follows:

$$\begin{aligned} Z &\sim N(0,\sigma_z^2) \\ S &\sim N(0,\sigma_s^2) \\ E_a &\sim N(0,\sigma_a^2) \\ E_n &\sim N(0,\sigma_i^2) \end{aligned} \quad (16)$$

The composite random variables are as follows:

$$\begin{aligned} X &= S + E_a \\ I_n &= S + E_n \\ I_i &= NS + E_i \\ I_{in} &= (N-1)S + E_{in} \end{aligned} \quad (17)$$

where

$$E_i = \sum_{n=1}^{N} E_n \sim N(0,N(\sigma_i^2))$$

$$I_{in} = I_i - I_n$$

$$E_{in} = E_i - E_n \sim N(0,(N-1)(\sigma_i^2))$$

III. Speculative Equilibrium

Each speculator is assumed to incur some cost (possibly zero) of entering the market and of collecting information. However, once the cost of entry or information collection has been incurred, the cost is a sunk cost and has no direct influence on short-term speculative behavior. Since costs are sunk, short-term profit-maximizing behavior is equivalent to maximizing revenues. Since speculators are risk neutral, each speculator maximizes expected revenue. We assume a Nash equilibrium in which each speculator (whether informed or uninformed) maximizes revenue (after costs are sunk), given the assumed behavior (demand functions) of all other speculators.

Since all uninformed speculators are identical, it seems reasonable to assume a symmetric equilibrium in which all uninformed speculators behave identically. Since all informed speculators are essentially identical (though they may possess different information), it is likewise reasonable to assume a symmetric equilibrium in which all informed speculators behave identically (except for informational differences). The first-order conditions for speculator revenue maximization are given in Appendix A.

The only requirement of the second-order conditions is that the residual supply curve facing each speculator must slope upwards ($B_1>0$ and $B_2>0$). Otherwise, it would be possible to buy futures at infinitely low prices, or to sell futures at infinitely high prices. Thinking in terms of long run equilibrium, this suggests that the residual supply curve facing any potential entrant must also slope upward ($B_0>0$), lest there be an infinite temptation for new speculators to enter the market.

We compute the symmetric market equilibrium by assuming that the optimally derived parameters for the individual speculators are identical to the assumed parameter values for all other speculators. This is done by appropriate substitution of variables into the first-order conditions. The resulting equations do not appear to be solvable analytically, but they can be solved numerically using a computer. Using Newton's method, it is usually possible to obtain a convergence to an economically sensible equilibrium by starting with initial values of $B_i=100$ and $B_u=0$. (See Appendix B for the equations for a symmetric speculative equilibrium.)

Holding N constant, an increase in M causes R_i and R_u to decrease, while R_i/R_u increases. In the limit, as M approaches infinity, R_u approaches zero while R_i approaches a finite positive value, which is proportional to the amount of noise. As M approaches infinity, A_u and B_u decline in inverse proportion to M, while R_u declines in inverse proportion to the square of M. As M approaches infinity, each of the uninformed speculators becomes an infinitismal player in the market. The assumption that $M=\infty$ is consistent with assuming $C_u=R_u=0$. (See Appendix C for the equilibrium equations with zero-cost entry by uninformed speculators.)

When there are only a finite number of uninformed speculators, the market price is biased downward. However, in the model with an infinite number of uninformed speculators, the average price which occurs in the marketplace is unbiased compared with the average price which solves the fundamentals equation (3) for $P_1=P_2$ when $X=0$. The model with zero-cost entry by uninformed speculators is simpler and provides an ideal case against which to judge the efficiency of financial markets.

IV. Welfare Losses

Let $L(P_1,P_2)$ be the deadweight loss to society from misprediction of the future price of the commodity. When the future price is underpredicted ($P_1<P_2$), farmers underproduce the commodity. When the future price is overpredicted ($P_1>P_2$), farmers overproduce the commodity. Only when price is perfectly predicted does deadweight loss equal zero. The formula for deadweight loss is:

$$L(P_1,P_2) = (P_2-P_1)^2\left(\frac{B_sB_d}{2B_{sd}}\right) \tag{18}$$

Substituting for P_2 from (12) into (18) we obtain:

$$L(P_1) = P_1^2\left(\frac{B_sB_{sd}}{2B_d}\right) - P_1\left(\frac{A_{ds}B_s}{B_d}\right) - P_1X\left(\frac{B_s}{B_d}\right) + X^2\left(\frac{B_s}{2B_dB_{sd}}\right) + X\left(\frac{A_{ds}B_s}{B_dB_{sd}}\right) + \left(\frac{A_{ds}^2B_s}{2B_dB_{sd}}\right) \tag{19}$$

Suppose no information at all is known about X. In that case the best *ex ante* prediction of price is $P_1=P_0$, where $P_0=A_{ds}/B_{sd}$. Substituting into (19) and taking expectations, the deadweight loss is:

$$L_x = L(P_1=P_0) = (\sigma_s^2+\sigma_a^2)\left(\frac{B_s}{2B_dB_{sd}}\right) \tag{20}$$

Suppose instead that the signal S is perfectly observed. In that case the best predicted price is $P_1=P_s$, where $P_s=(A_{ds}+S)/B_{sd}$. Substituting into (19) and taking expectations, the deadweight loss is:

$$L_s = L(P_1=P_s) = (\sigma_a^2)\left(\frac{B_s}{2B_dB_{sd}}\right) \tag{21}$$

Suppose that the signal S is not perfectly observed, but that the combined information of N speculators is perfectly known. In that case the optimal prediction of price is $P_1=P_o$, where

$$P_o = A_{ds}/B_{sd} + \frac{(\sigma_s^2)I_i}{(\sigma_s^2+\sigma_i^2/N)B_{sd}N} \tag{22}$$

Substituting into (19) and taking expectations, the deadweight loss is: (23)

$$L_o = L(P_1=P_o) = \left[\frac{\sigma_s^2\sigma_i^2}{(\sigma_s^2)N+(\sigma_i^2)} + \sigma_a^2\right]\left[\frac{B_s}{2B_dB_{sd}}\right]$$

Suppose instead that the market price (P_m) is the price actually used, where P_m is derived from the model in Section V. Substituting $P_1=P_m$, where P_m is derived from (9) and (C6) to

(C10), the expected deadweight loss is:

$$L_m = L(P_1=P_m) = (\sigma_s^2(1+T_8)+\sigma_a^2)\left(\frac{B_s}{2B_dB_{sd}}\right) \tag{24}$$

where $$T_8 = \frac{(k-1)[-N^2k^2+5N^2k-6N^2-2Nk^2t-2Nk^2+8Nkt+4Nk-6Nt-tk^2+tk]}{(k-2)[Nk+k-2N+2kt-2t]^2} \tag{25}$$

In the special case where t=1, $T_8=-N/(N+1)^2$.

The above calculations of L_m and L_o implicitly assume that there is zero resource cost for a speculator to become informed. More realistically, assume $C_i>0$, where C_i is the cost for a speculator to become informed. In that case, the total welfare loss from the futures market institution is:

$$WL_m = L_m(N=N_m) + N_mC_i, \tag{26}$$

where N_m is the number of informed speculators in the market. The total welfare loss from an optimal forecasting institution is:

$$WL_o = L_o(N=N_o) + N_oC_i, \tag{27}$$

where N_o is the optimal number of informed forecasters. For purposes of this paper, we do not ask whether this optimal forecasting institution is actually feasible.[3] It is merely a basis for comparison.

In the optimal forecasting institution, the optimal number of forecasters, N_o, is easily determined by taking derivatives of (27) with respect to N_o. This yields the following quadratic equation in N_o:

$$N_o^2(\sigma_s^4) + 2N_o(\sigma_s^2)(\sigma_i^2) + \sigma_i^4 - (\sigma_s^4)(\sigma_i^2)B_s = 0 \tag{28}$$

$$\overline{}$$
$$2B_d B_{sd} C_i$$

Only the positive root of this equation is economically relevant. We may similarly compute the optimal number of informed speculators in the market by taking the derivative of (26) with respect to N_m. This yields a complex expression which is not reproduced here.

V. Additional Welfare Losses

The previous section computed welfare losses under the assumption that price forecasting is the only function performed by the futures market. In reality, the futures market also provides income insurance to risk-averse farmers who wish to hedge their supply decisions against price uncertainty. Performance of this insurance function is presumably necessary for the existence of a futures market, since it is only hedgers who rationally accept the losses which allow speculators to earn the positive revenues needed to defray their costs of information collection.

The basic model in Section II specified an exogenous amount of noise trading, having mean zero and variance σ_z^2. In the model, if there is no noise trading, speculators are unable to earn any positive revenues which can defray their costs. Noise trading is necessary for the existence of the market.

The source of this noise trading was not explained. If we imagine the existence of a separate class of traders, called "noise traders," who trade quantities completely at random, without regard to price, this would be one possible interpretation of the model. Under this interpretation, noise trading is completely irrational, since the noise traders necessarily incur losses on average.

However, other explanations of noise trading, more consistent with notions of economic rationality and rational expectations, are possible. One explanation is that the noise is caused by uncertainty regarding the numbers of informed and uninformed speculators operating in the market. The basic model assumes that these numbers are known and fixed, but in reality these numbers vary and are not precisely known. Since speculators are assumed to earn zero economic profits in equilibrium, it should be a matter of indifference to individual speculators either to enter the market or to stay out. Hence, it is neither rational nor irrational for individual speculators to exhibit random behavior with respect to entry and exit decisions.

Other possible explanations include incomplete knowledge among speculators about the optimal strategies to pursue. This may lead to different speculators pursuing different strategies, which add another element of randomness. Speculators may also have limited wealth, and may choose to divide their investments among activities in several markets. Changes in perceived profit opportunities in alternative markets will affect the amount of wealth invested in a particular market, and may do so in a manner that is not easily predicted by market participants. Total farm supply and the amount of hedging desired by farmers may also be subject to uncertainty, and thereby create noise in the market.

Regardless of the source of the noise, those who create noise by trading randomly necessarily incur losses, unless those losses are somehow transferred to other market participants. Even if speculators themselves are the source of the noise, rational speculators will not rationally accept the losses which result from noise trade. If, by hypothesis, such losses were to be incurred, we would expect speculators either to exit the market until losses were no longer incurred or, if remaining in the market, to reduce their offered prices to purchase futures. Either way, the prices obtained by farmers who sell futures is reduced.

Farmers, being hedgers, rationally accept such losses on the futures price relative to the expected future spot price. It is only because hedgers accept such losses that speculators can find it rational to engage in costly speculation. This should not be interpreted as a statement that there are no irrational speculators. Empirically speaking, there may be irrationality, perhaps even a great deal of irrationality. However, it is not theoretically necessary to assume irrationality as a necessary cause of noise trading and consequent source of speculator incomes.

Adopting the hypothesis that speculators do not suffer chronic losses, but instead shift those losses onto hedgers, we can proceed with further application of the basic model. The variance of noise is specified as an exogenous parameter of the model. There is no necessary relationship between the amount of trading noise and the deadweight loss; nor is there any necessary relationship between the amount of trading noise and the level of risk aversion of the hedgers. If we rule out repeat trading of futures contracts, then the amount of noise should be less than the volume of spot trade. Beyond this, one cannot state theoretically how much noise there will be.

If the losses caused by noise trading are too large, hedgers may choose not to sell futures. Such action would eliminate the profit for informed speculation, and render the particular futures market infeasible. Despite this fact, there is no mechanism which would cause traders to trade less noisily. Hence, for particular parameter values, there may be no futures market. For other parameter values, the market might be thinly traded.

The strategy for applying the basic model is to assume a set of parameters for the market, compute the amount of losses from noise trading (which equal speculator incomes), impose those losses on hedgers in the form of reduced prices for purchasing futures, and then calculate the amount (if any) which hedgers are willing to sell. Such a model is worked out in Appendix D.

VI. Value Marginal Products

Samuelson's remarks, discussed earlier, suggest both equity and efficiency concerns with respect to the question of whether market speculators are paid less than or more than their Value Marginal Product (VMP). There are several different ways of defining VMP, only some of which are relevant here.

VMP can be calculated with respect to the extensive margin (how many speculators become informed), the intensive margin (how much time, money, or effort is spent by each speculator to become informed), and, in the case of information, a utilization margin (how well does the market utilize the information). Since this paper does not analyze variations in effort or information precision of particular speculators, except for the discontinuous decision to become informed, I will only calculate VMP's with respect to the extensive margin. VMP on the extensive margin tells us whether too many or too few speculators enter the market.

VMP on the extensive margin can be measured in a number of ways. First we can ask whether the measure should be short run, medium run, or long run. A short-run measure would tell us what happens if the number of informed speculators changes from its long-run equilibrium value, but other speculators are unaware of the change, and consequently do not alter their strategies in response. A medium-run measure would tell us what happens if the number of informed speculators changes from its long-run equilibrium, other speculators become aware of the change, and consequently adjust their strategies (but not their numbers) in response. A long-run measure would tell us what happens if the number of informed speculators deviates from its long-run equilibrium, time is permitted for further adjustments in the number of informed speculators, and consequently a new long-run equilibrium is established.

Finally, we can ask whether the VMP measure should be based on a unit change in the number of informed speculators, or whether the measure should be based on an infinitesimal change in the number of speculators. If a unit change is used, we must ask whether we compare a situation of N speculators with a situation of N-1 speculators, or whether we compare N speculators with N+1 speculators. No matter which choice of measure is used, we compute VMP's by ascertaining the change in deadweight loss which occurs when we change the number of informed speculators.

For purposes of studying Samuelson's conjecture, the best measure of VMP on the extensive margin compares the situation with N informed speculators with N-1 informed speculators. If the reduction in deadweight loss (L_o or L_m) in moving from N-1 to N informed speculators exceeds the cost of an informed speculator (C_i), then society should allow or cause the Nth speculator to enter the market. Contrariwise, if the cost of the speculator exceeds the reduction in deadweight loss, then society should not hire the Nth speculator.

Define VMP_m as the medium-term VMP of a speculator working within the futures market institution:

$$VMP_m = L_m(N=N_m-1) - L_m(N=N_m) \qquad (29)$$

If $R_i=VMP_m$, this is an indication of market efficiency on the extensive margin, but is not necessarily an indicator of overall efficiency. If R_i differs VMP_m, one could argue for a tax or subsidy on speculator earnings, for both equity and efficiency reasons, to eliminate the differential.

If the main concern is equity, VMP can be viewed as a measure of a person's economic contribution to society, according to which a person ought to be paid. A natural question to ask is, "If the person did not exist, by how much would total output decline?" The medium-run VMP tells us the marginal value of output which the Nth speculator currently contributes to the forecasting task.

In the long run, if the Nth speculator did not exist, the incentives of the market-place would attract replacement speculators. If all speculators face identical costs of obtaining information, then the cost of replacement is C_i. The long-run market equilibrium is identical to what it was before, but resources in the amount of C_i have been attracted away from other productive activities. Hence, social output is reduced by C_i, and the long-run VMP of the speculator is C_i.

Given free entry of speculators into the market, we expect speculators to enter the market until $R_i=C_i$. The only reason for R_i to exceed C_i is if a) entry is somehow blockaded, or b) there are integer constraints which prevent R_i from exactly equalling C_i, because $R_i=C_i$ requires a fractional number of speculators.[4] The problem posed by b) is unlikely to be very serious, unless the equilibrium has only a small N. The problem posed by a) is unlikely to occur, so long as there is "equal opportunity" speculation. Note that lack of "equal opportunity" in speculation would have both efficiency and equity implictions.

VII. Computations of VMP's and Welfare Losses

This section reports results of a computer simulation to compute possible ratios of deadweight loss and VMP. No particular assumption is made about the relative lengths of period 1 and period 2 and no effort is made to make period 1 arbitrarily short. Since the tested parameters are not based on known empirical values, the simulation should only be taken to suggest a range of theoretical possibilities for the extent of cost-inefficiency of futures markets.

Inspection of (20), (21), (23), and (24) indicates that deadweight losses are unaffected by the parameters A_s, A_d, and σ_z. The parameters B_s and B_d affect the deadweight losses in obvious ways which require no computer simulation to determine. Inspection of (C11) indicates that R_i is unaffected by A_s, A_d, and B_s. The parameters B_d and σ_z affect R_i in obvious ways which require no computer simulation to determine. The parameter σ_a has a constant effect on the deadweight losses and no effect on R_i. We can without loss of generality set $\sigma_a=0$. If speculators had been modeled as risk averse, rather than risk neutral, σ_a might well have had some influence on market outcomes.

In the computer model, I normalize $B_d=\sigma_s=1$ and normalize $A_s=A_d=\sigma_a=0$. I allow σ_i^2 and σ_z^2 to vary independently through the range of values: .0001, .001, .01, .1, 1, 10, 100, 1000, 10000.

I also allow N_m to vary through the range of values: 3, 10, 30, 100, 300, 1000. I allow Bs to vary through the range of values: .04, .2, 1, 5, 25. This means that 9x9x6x5 = 2,430 cases were tested. Because of the integer constraint, we must assume that C_i lies somewhere between $R_i(N=N_m)$ and $R_i(N=N_m+1)$. In my calculations I assumed that $C_i=R_i(N=N_m+.5)$.

Given the exogenous parameters, the model computes the endogenous behavioral parameters, B_i, B_a, G_i, A_i, and A_a, and computes the various outcomes, R_i, L_m, WL_m, VMP_m, etc. The model also computes the optimal number of forecasters, N_o, in the optimal forecasting institution. If this number is greater than 2, I ignore integer constraints and simply use whatever fractional number is computed. If the computed N_o is less than 2, I specifically determine whether the optimal number of forecasters is 0, 1, or 2. Frequently, the optimal number is zero.

In order to compute the efficiency of the futures market, it is necessary to have some measure of "efficiency." One such measure is:

$$RL_1 = \frac{(WL_m - WL_o)}{(WL_o - L_s)} \tag{30}$$

This ratio tells us the lost welfare of the futures market as a multiple of the lost welfare of the optimal forecasting institution. This ratio of losses can never be less than zero, which implies perfect efficiency. In the range of parameters sampled, RL_1 varied from .00017 to 1799, with a mean of 39.87.

The median was 3.601, and the interquartile range was .8152 to 22.98. Depending on what sets of parameter values are empirically likely, the financial markets might be either fairly efficient, or grossly inefficient.

Another loss ratio of possible interest is the following:

$$RL_2 = \frac{(WL_m - L_x)}{(L_x - L_s)} \tag{31}$$

This ratio of losses tells us the welfare loss of the futures market relative to the difference in deadweight loss (L_x-L_s) between having no information and having perfect knowledge of the signal S. This ratio can never be less than negative one. If the ratio is greater than zero, this means that the futures market has made society worse off than if society had spent no resources at all in collecting information. This could occur, for instance, if the futures market encouraged too much expensive information gathering relative to the value of that information to society. If the ratio is less than zero, then society is better off with a futures market.

For the range of parameters sampled, RL_2 ranged from -1 to 1799. The median was -.007152 and the interquartile range was -.9331 to 3.078. In nearly half (49%) of the cases RL_2 was greater than zero, indicating that having a futures market is worse than collecting no information at all. The 10th percentile was 30.77 and the 5th percentile was 89.70. The mean value of RL_2 in the sample was 23.42. In other words, having a futures market is, on average in this sample, at least twenty times worse than having no futures market at all.

The denominator of RL_2, (L_x-L_s), varies in the sample only when B_s varies. The mean value of RL_2 rises as B_s falls. This mean value is 2.797 when $B_s=25$, 3.328 when $B_s=5$, 5.985 when $B_s=1$, 19.27 when $B_s=.2$, and 85.71 when $B_s=.04$. As B_s approaches zero ($B_s=0$ implies vertical supply curve), RL_2 approaches infinity, since the social value of information approaches zero, yet the futures market spends costly resources to acquire it.

Another factor which significantly influences the value of RL_2 is σ_z^2. As σ_z^2 rises, the calculated value of RL_2 also rises. When $\sigma_z=.01$ the mean value of RL_2 in the sample is -.6432. When $\sigma_z=1$ the mean value of RL_2 is 0.824, and when $\sigma_z=100$ the mean value of RL_2 is 147.5. Obviously, empirical work is needed to determine what ranges of parameter values are realistic. Theory alone can not offer sanguine conclusions about the informational cost efficiency of financial markets.

It may be useful to consider the possible sources of welfare loss. If we switch from the futures market to the optimal forecasting institution, keeping constant for the number of forecasters, we can define a new loss ratio:

$$RL_3 = \frac{[L_m(N=N_m) - L_o(N=N_m)]}{[WL_m - WL_o]} \tag{32}$$

The median value of RL_3 is .0081, the interquartile range is .0005 to .13, and the full range is .000000065 to .9999. Since RL_3 is usually small, this implies that most of the welfare loss usually comes from attracting the wrong number of informed speculators into the market. Generally speaking, the "wrong number" turned out to be too high rather than too low. In over 85% of the cases sampled, informed speculators were paid more than their medium-run VMP. The ratio R_i/VMP_m ranged from .0115 to 7,693,000, with a median value of 69.9 and an interquartile range of 4.02 to 1,041.

RL_3 consists of two components, the loss due to noise (the variation in Z) and the loss due to suboptimal values of G_i. Both losses result from inefficient utilization of information relative to what was available to the market. The proportion of loss contributed by noise ranged from 50% to 100% of RL_3, and conversely, the proportion contributed by bias in G_i ranged from 0% to 50%. In a model with finite M, downward bias in average price would be an additional source of deadweight loss.

VIII. Summary and Conclusions

In a model with risk-neutral speculators and noncollusive imperfect competition with consistent conjectures and rational expectations, it was shown a) that there is no incentive to gather costly information unless the market has exogenous noise, b) that the ratio of speculators' compensation to their VMP increases with the amount of noise, c) that the ratio of speculators' compensation to their VMP increases as the time interval during which private information is held is reduced, and d) that financial markets are inefficient with respect to the costs incurred in gathering information. Conclusions (a) and (b) are unsurprising, given the models and reasoning in Grossman and Stiglitz (1980) and Kyle (1989). Conclusions (c) and (d) are specific to this paper and tend to support Samuelson's conjecture.

Given that speculators are frequently paid either more or less than their medium-run VMP, and there appears to be no mechanism which assures equality, it is not surprising that suboptimal numbers of informed speculators are the main cause of market inefficiency. Another cause of inefficiency, which runs a distant second, is the randomness of price which is caused by market noise. A third source of inefficiency is the inefficient utilization of the information which is available to informed speculators. A fourth source of inefficiency, the magnitude of which was not measured in this paper, is the downward bias of average market price, whenever there are only a finite number of uninformed speculators in the market.

One possible drawback of the present model is that "noise trading" is exogenously specified as an independent parameter of the model. An increase in noise trading increases the profits to informed speculators, but it also increases the losses to noise traders. At some point, the potential losses from noise trading must impose some constraint on the amount of noise trading which may occur. This in turn would limit the size of the "fortunes" which might be earned from informed speculation. An empirical examination of the extent of noise trading and other parameters is necessary to fully answer the questions raised by Samuelson's conjecture.

Appendix A. Behavior of Speculators

The revenue of the uninformed speculator is given by:

$$R_m = Q_m(P_2-P_1) \tag{A1}$$

Assume that all other speculators behave according to (4) and (5), but that one particular speculator behaves according to:

$$Q_m = A_m - B_m P_1 \tag{A2}$$

The values of P_1 and (P_2-P_1) are obtained from equations (9) and (14), provided we substitute:

$$A_0 = A_1 + A_m$$

$$B_0 = B_1 + B_m \tag{A3}$$

where

$$A_1 = -A_s + (M-1)A_u + NA_i$$

$$B_1 = B_s + (M-1)B_u + NB_i$$

Making all appropriate substitutions and taking expectations, we obtain:

$$\begin{aligned} R_m = {} & A_m A_{ds}/B_d - A_m B_{sd} A_0/(B_0 B_d) - B_m A_{ds} A_0/(B_0 B_d) \\ & - B_m G_i N \sigma_s^2/(B_0 B_d) + B_m B_{sd} A_0^2/(B_0^2 B_d) \\ & + B_m B_{sd}[\sigma_m]/(B_0^2 B_d) \end{aligned} \tag{A4}$$

where $\sigma_m = \sigma_z^2+G_i^2N^2\sigma_s^2+G_i^2N\sigma_i^2$ (A5)

The uninformed speculator must choose A_m and B_m to maximize R_m in (A4). The first-order conditions are:

$$\begin{aligned} (dR_m/dA_m) = {} & A_{ds}/B_d - B_{sd}A_0/(B_0 B_d) \\ & - A_m B_{sd}/(B_0 B_d) - B_m A_{ds}/(B_0 B_d) \\ & + 2B_m B_{sd} A_0/(B_0^2 B_d) = 0 \end{aligned} \tag{A6}$$

$$\begin{aligned} (dR_m/dB_m) = {} & A_m B_{sd} A_0/(B_0^2 B_d) \\ & + B_m A_{ds} A_0/(B_0^2 B_d) - A_{ds} A_0/(B_0 B_d) \end{aligned} \tag{A7}$$

$$- G_iN(\sigma_s^2)/(B_0B_d) + B_mG_iN(\sigma_s^2)/(B_0^2B_d)$$

$$+ B_{sd}A_0^2/(B_0^2B_d) - 2B_mB_{sd}A_0^2/(B_0^3B_d)$$

$$+ B_{sd}[\sigma_m]/(B_0^2B_d) - 2B_mB_{sd}[\sigma_m]/(B_0^3B_d) = 0$$

If we multiply (A6) by $(B_0^2B_d)$ and simplify, we obtain:

$$0 = A_{ds}(B_1+B_m)B_1 + B_{sd}(A_1B_m-A_1B_1-2A_mB_1) \quad (A8)$$

If we multiply (A6) by A_0/B_0 and add to (A7), multiply the result by $(B_0^3B_d)$, and simplify, we obtain:

$$0 = -G_iN(\sigma_s^2)(B_1+B_m)B_1 + B_{sd}[\sigma_m](B_1-B_m) \quad (A9)$$

The revenue of the informed speculator is given by:

$$R_n = Q_n(P_2-P_1) \quad (A10)$$

Assume that all other speculators behave according to (4) and (5), but that one informed speculator behaves according to:

$$Q_n = A_n - B_nP_1 + G_nI_n \quad (A11)$$

The values of P_1 and (P_2-P_1) are obtained from equations (9) and (14), provided we substitute:

$$A_0 = A_2 + A_n$$

$$B_0 = B_2 + B_n \quad (A12)$$

$$G_iI_i = G_iI_{in} + G_nI_n$$

where $A_2 = -A_s + MA_u + (N-1)A_i$

$B_2 = B_s + MB_u + (N-1)B_i$

Making all appropriate substitutions and taking expectations, we obtain:

$$R_n = A_nA_{ds}/B_d + G_n(\sigma_s^2)/B_d - B_nA_{ds}A_0/(B_0B_d) \quad (A13)$$

$$- B_n[(N-1)G_i+G_n](\sigma_s^2)/(B_0B_d) - A_nB_{sd}A_0/(B_0B_d)$$

$$- G_nB_{sd}[(\sigma_s^2)((N-1)G_i+G_n)+(\sigma_i^2)G_n]/(B_0B_d)$$

$$+ B_nB_{sd}A_0^2/(B_0^2B_d) + B_nB_{sd}[\sigma_n]/(B_0^2B_d)$$

where $\sigma_n = \sigma_z^2 + (\sigma_s^2)[G_i^2(N-1)^2+2G_iG_n+G_n^2]$ (A14)

$+ (\sigma_i^2)[G_i^2(N-1)+G_n^2]$

The first-order conditions for the informed speculator are:

$$(dR_n/dA_n) = A_{ds}/B_d - B_nA_{ds}/(B_0B_d) - B_{sd}A_0/(B_0B_d) - A_nB_{sd}/(B_0B_d) + 2B_nB_{sd}A_0/(B_0^2B_d) = 0 \quad \text{(A15)}$$

$$(dR_n/dB_n) = - A_{ds}A_0/(B_0B_d) + B_nA_{ds}A_0/(B_0^2B_d) - [(N-1)G_i+G_n](\sigma_s^2)/(B_0B_d) + B_m[(N-1)G_i+G_n](\sigma_s^2)/(B_0^2B_d) + A_nB_{sd}A_0/(B_0^2B_d) + B_{sd}A_0^2/(B_0^2B_d) - 2B_nB_{sd}A_0^2/(B_0^3B_d) + G_nB_{sd}[((N-1)G_i+G_n)(\sigma_s^2)+G_n(\sigma_i^2)]/(B_0^2B_d) + B_{sd}[\sigma_n]/(B_0^2B_d) - 2B_nB_{sd}[\sigma_n]/(B_0^3B_d) = 0 \quad \text{(A16)}$$

$$(dR_n/dG_n) = (\sigma_s^2)/B_d - (\sigma_s^2)B_n/(B_0B_d) - B_{sd}G_n[\sigma_s^2+\sigma_i^2]/(B_0B_d) - B_{sd}[(N-1)G_i+G_n](\sigma_s^2)+G_n(\sigma_i^2)/(B_0B_d) + 2B_nB_{sd}\{[(N-1)G_i+G_n](\sigma_s^2)+G_n(\sigma_i^2)\}/(B_0^2B_d) = 0 \quad \text{(A17)}$$

If we multiply (A15) by $(B_0^2B_d)$ and simplify, we obtain:

$$0 = A_{ds}(B_2+B_n)B_2 + B_{sd}(A_2B_n-A_2B_2-2A_nB_2) \quad \text{(A18)}$$

If we multiply (A15) by A_0/B_0 and add to (A16), multiply the result by $(B_0^3B_d)$, and simplify, we obtain:

$$0 = -[(N-1)G_i+G_n](\sigma_s^2)(B_2+B_n)B_2 + B_{sd}G_n\{[(N-1)G_i+G_n](\sigma_s^2)+G_n(\sigma_i^2)\}(B_2+B_n) + B_{sd}[\sigma_n](B_2-B_n) \quad \text{(A19)}$$

If we multiply (A17) by $(B_0^2B_d)$ and simplify, we obtain:

$$0 = (\sigma_s^2)(B_2+B_n)B_2 - B_{sd}G_n(B_2+B_n)[\sigma_s^2+\sigma_i^2] - B_{sd}(B_2-B_n)\{[(N-1)G_i+G_n](\sigma_s^2)+G_n(\sigma_i^2)\} \quad \text{(A20)}$$

Appendix B. Symmetric Speculator Equilibrium

In order to compute the symmetric market equilibrium, we impose symmetry by substituting $A_m=A_u$, $B_m=B_u$, $A_n=A_i$, $B_n=B_i$, and $G_n=G_i$ into equations (A8), (A9), (A18), (A19), and (A20), and let $R_u=R_m$ and $R_i=R_n$. Inspection of equations (A8) and (A18) indicate that A_u and A_i are computable as linear functions, given knowledge of B_u, B_i and the basic parameters. These equations can be rearranged to produce the linear system:

$$A_u[MB_0+B_0-2MB_u] + A_i[NB_0-2NB_u] = (A_{ds}/B_{sd})B_0(B_0-B_u) + A_s(B_0-2B_u) \quad \text{(B1)}$$

$$A_u[MB_0-2MB_i] + A_i[NB_0+B_0-2NB_i] = (A_{ds}/B_{sd})B_0(B_0-B_i) + A_s(B_0-2B_i) \quad \text{(B2)}$$

Similarly, inspection of (A20) indicates that G_i is also computable as a simple function, given knowledge of B_u, B_i, and the basic parameters. This equation can be rearranged as:

$$G_i = \frac{(B_0-B_i)B_0}{B_{sd}[(NB_0+B_0-2NB_i)+2(B_0-B_i)(\sigma_i^2/\sigma_s^2)]} \quad \text{(B3)}$$

This leaves B_u and B_i as crucial behavioral parameters which must be computed from the basic parameters. Substituting for G_i from (B3) into (A9) and (A19), and simplifying, we obtain:

$$-(\sigma_s^2)N(B_0-B_i)[B_0+(N-1)B_u-NB_i] - (\sigma_i^2)N(B_0-B_i)(B_0-B_i) + (B_0-2B_u)Z_x = 0 \quad \text{(B4)}$$

$$-(\sigma_i^2)(N-1)(B_0-B_i)(B_0-B_i) + (B_0-2B_i)Z_x = 0 \quad \text{(B5)}$$

where $Z_x=(\sigma_z^2)B_{sd}^2[(NB_0+B_0-2NB_i)+2(B_0-B_i)(\sigma_i^2/\sigma_s^2)]^2/(B_0^3)$

If we solve for Z_x from (B4) and (B5) and equate, we obtain the following additional relationship:

$$(\sigma_s^2)N(B_0+(N-1)B_u-NB_i)(B_0-2B_i) \quad \text{(B6)}$$
$$+ (\sigma_i^2)(B_0+2(N-1)B_u-2NB_i)(B_0-B_i) = 0$$

Appendix C. Zero-Cost Entry by Uninformed Speculators

Define aggregate variables, A_a and B_a, as follows:

$$A_a = MA_u \quad \text{and} \quad B_a = MB_u \quad \text{(C1)}$$

Equations (B1), (B2), (B4), and (B6) become:

$$A_a[B_0] + A_i[NB_0] \quad \text{(C2)}$$
$$= (A_{ds}/B_{sd})B_0(B_0) + A_s(B_0)$$

$$A_a[B_0-2B_i] + A_i[NB_0+B_0-2NB_i] \quad \text{(C3)}$$
$$= (A_{ds}/B_{sd})B_0(B_0-B_i) + A_s(B_0-2B_i)$$

$$-(\sigma_s^2)N(B_0-B_i)[B_0-NB_i] \quad \text{(C4)}$$
$$-(\sigma_i^2)N(B_0-B_i)(B_0-B_i) + (B_0)Z_x = 0$$

$$(\sigma_s^2)N(B_0-NB_i)(B_0-2B_i) \quad \text{(C5)}$$
$$+ (\sigma_i^2)(B_0-2NB_i)(B_0-B_i) = 0$$

Equations (B3) and (B5) remain the same.

Define $k=B_0/B_i$. Substituting $B_0=kB_i$ into (C5) and dividing by B_i^2, we obtain a quadratic equation in k. Define $t=\sigma_i^2/\sigma_s^2$. The solution for k is:

$$k = \frac{N+2}{2} + \frac{(N+1)t}{2(N+t)} + \frac{SQR\{[(N-2)(N+t)+(N-1)t]^2+8(N-1)t(N+t)\}}{2(N+t)} \quad \text{(C6)}$$

By substituting $B_0=kB_i$ into (B5), we can show:

$$B_i = \frac{\sigma_z B_{sd} Y}{\sigma_i} \tag{C7}$$

where $$Y = \frac{(Nk+k-2N+2kt-2t)SQR(k-2)}{(k-1)kSQR(k(N-1))} \tag{C8}$$

From (B3) we can then show:

$$G_i = \frac{B_i SQR(k-2)}{B_{sd}Y\ SQR(k(N-1))} \tag{C9}$$

From (C2), (C3), and (A3) we can derive:

$$A_a = (A_{ds}/B_{sd})(k-N)B_i + A_s \tag{C10}$$

$$A_i = (A_{ds}/B_{sd})B_i$$

$$A_0 = (A_{ds}/B_{sd})kB_i$$

From (A13) we can derive:

$$R_i = \frac{\sigma_s^2\sigma_z[k^3(N+1+2t)+k^2(-N2-6N-2Nt-2-7t) +k(5N^2+8N+8Nt+5t)+(-6N^2-6Nt)]}{B_d k^3 Y\sigma_i(N-1)(k-1)} \tag{C11}$$

It is noteworthy that B_i, G_i, A_i, B_0, A_0, and R_i are all proportional to σ_z. For the special case where t=1 (σ_i=σ_s), it can be shown:

$$B_i = \frac{\sigma_z B_{sd} SQR(N+1)}{\sigma_i N} \tag{C12}$$

$$B_0 = (N+1)B_i \tag{C13}$$

$$G_i = \frac{NB_i}{B_{sd}(N+1)} \tag{C14}$$

$$A_i = (A_{ds}/B_{sd})B_i \tag{C15}$$

$$A_0 = (N+1)(A_{ds}/B_{sd})B_i \tag{C16}$$

From (9) we can (by setting Z=0 and I_i=0) determine the average price which occurs in the marketplace. This price is A_0/B_0, which equals A_{ds}/B_{sd} in the model here. On average, this price is unbiased compared with the average price (P_0) which solves the fundamentals equation (3) for P_1=P_2 when X=0.

FOOTNOTES

1. See, for example, Silver (1989, p. 75) and Kirzner (1978).

2. Critics of the efficiency of financial markets include: Shiller (1981) who claims that financial markets are subject to excess volatility, DeBondt and Thaler (1985) who claim that markets overreact to new information, De Long, et al (1989) who estimate substantial reductions in the stock of equity capital due to noise trading (p. 695), and Shleifer and Vishny (1990) who claim that traders concentrate too much attention on predicting short-term price changes rather than long-term fundamentals. Stock market crashes also suggest a rejection of any extreme form of rational expectations model. This paper makes a separate and independent criticism with respect to the cost efficiency of information acquisition.

3. See Lundgren (1994) for a description of forecasting incentives based on VMP.

4. If effort per speculator is elastic (unlike in the present model), it may well be that even small N would not result in significant speculator rents, absent collusion.

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